APPLICATION OF MATHEMATICAL MODELLING TO DIESEL-FUELLED ENERGY EMISSION

By

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A Thesis submitted in fulfilment of the requirements for the degree of Master of Science in Applied Mathematics, Faculty of Health and Applied Sciences, Namibia University of Science and Technology, Windhoek, Namibia

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I, Philipus Nghifikepunye Nangolo, hereby state that the work contained in the thesis entitled "Application of Mathematical Modelling to Diesel Fuelled Energy Emission" is my own original work and that I have not previously, in its entirety or in part, submitted it at any university or other higher education institution for the award of a degree.

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DEDICATION

"I dedicate this thesis to my mum, Sabina Gotlieb"

ABSTRACT

In this thesis the Gaussian plume model is proposed as a method for solving problems related to the transportation of pollutants due to advection by wind and turbulent diffusion. The idea of advection and diffusion is fundamental to this thesis as well as its mathematical derivations from the initial principles to the explanation of the governing partial differential equation. Dimensional analysis technique has been employed as well as Fick's first and second law of diffusion. The concentration distribution of a pollutant freed into the atmosphere may be defined by the advection-diffusion equation which is a second-order partial differential equation of parabolic form.

Assumptions underlying the model in this thesis have been explained, their origin from advection diffusion equation and the important properties of the plume solution in the atmosphere. The results are then applied to solving problem in which emission rates are strongminded in a given power plant. We demonstrated our outcomes with an application for ongoing research of emission from diesel power plant (Anixas Power Plant, Walvis Bay).

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CHAPTER 1

GENERAL INTRODUCTION

1.1 Background of the study

When air pollution began to have a substantial consequence on human life, it became essential to learn and understand the links between emission sources and the air quality deterioration and associated health effects. In the past years, the dispersion of pollutants in the atmosphere has been the object of study and investigation in industrialized countries. The problems caused by air pollution are complex and affect natural processes, influencing the ecological balance. Only after the impacts of the sources have been assessed correctly will it be possible to devise and implement rational, convincing and effective policies to improve air quality (Arya, 1999).

In Namibia, the effects of fuel emissions from power plants have not been well considered. As a result, effective policies to shield human health and therefore, the atmosphere from such emissions are lacking. Therefore, it's vital that such emissions be scrutinized and modelled.

The Anixas diesel-fuelled power plant in the Republic of Namibia has been functioning since July 2011 to help in serving the height power necessities within the Namibian grid. The station is set adjacent to the prevailing Paratus Power Plant at Walvis Bay and it provides an emergency standby capacitance of 22.5 MW to the country. The 22.5 MW generation is shared among 3 caterpillar V16 cylinder diesel generator sets (Figure one.1). The generators use light-weight oil (LFO) for beginning and stopping and significant oil (HFO) for traditional operation. The LFO is so not expected to possess a major impact on the aerosolised and particle emissions.

Figure 1.1: Anixas Power Station: Oil-fuelled generator sets.

Each generator set is linked to an emissions analyser (Figure 1.2) measuring particulate and gaseous emissions.

Figure 1.2: Emissions analysers

A 60 metre high cluster exhaust stack (Figure 1.3) disperses the emissions high up reducing ambient concentration of any pollutant.

Figure 1.3: Anixas 60m high cluster exhaust stack

The key contaminants released from a diesel engine comprise Nitrogen oxides (NO_x, Sulphur dioxide (SO_2 and particulate material (soot and fuel ash). Emissions of CO and unburned Hydro Carbons (UHC) from diesel engines are insignificant for liquid fuel engine plants.

1.2 Thesis organisation

Chapter 1 contains an introduction about a brief overview of air pollution dispersion modelling and its significance to Namibia's energy production sector. Furthermore, it details the objectives of the project, the methodology used as well as the importance of the research.

Chapter 2 discuses the literature survey on research domain and presents a summary on a number of researches already conducted in relation to the research problem under consideration. The literature, viz thesis and publications specifically deal with emissions and air pollution dispersion in the atmosphere.

Chapter 3 describes the basics of any flow simulation are based on the conservation of various laws expressed in the form of equations. These equations are to be solved to get the solution. The transportation of fluid flow relies on solving four equations, viz. conservation of mass, advection, diffusion and also species concentration in case of combustion. This chapter discusses the equations used by the model.

Chapter 4 covers the fundamentals of Gaussian Plume Model - its evolution, assumptions, basic structure, behaviour and its area of application are covered in this chapter.

Chapter 5 covers the geometrical modelling and the inputs needed for the simulation of fluid flow in the atmosphere and emission process in diesel engine at the power plant are discussed in this section. The analysis of the data is also covered in this chapter.

In chapter 6, the performance and emission analysis with respect to the specified power plant and geometric variations using Gaussian Plume Model are discussed. Results were presented and more discussion was tickled in detail.

Chapter 7 gives a summary of the observations and interpretations made from the dissertation. The concluding statements are drawn, describing the achievements of the research work and the possible applications in the real world. Recommendations are made from for future work.

Last but not least, the appendix contains the whole description of all benchmark methodology of our Gaussian model as well as References

1.3 A model and its purpose

A model could be a straightforward manner of representing a posh state of affairs. It doesn't have to be compelled to take into account each and every element concerned. Betting on the matter that has to be solved solely the vital characteristics are mentioned. Most world issues are often simplified victimization mathematical modelling as some way to strain extraneous data. Whereas, the ensuing model typically can't be solved precisely, it ought to a minimum of be "as straightforward as possible". Several ecological issues can't be solved without mathematical modelling, since the wildlife is interconnected and it is impractical to live all the individual relationships.

Mathematical models are utilized by scientists, analysts and engineers so as to review the interactions of the system elements and predict their behaviour(Briggs, 1975).

1.4 Concepts and definitions

The term transport means, processes that change substances through the layer and atmosphere by physical means. As an equivalence to communication, transport is the method by which the letter goes from one site to a different (Fisher & Macqueen, 1981). The communicating truck is that the analogy for the fluid, and therefore the letter itself is that the analogy for the chemical species. The two main means of transport in environmental mechanics are temperature change (transport related to the movement of a fluid) and diffusion (transport related to accidental motions inside a fluid).

Similarly, the word transformation in this context, defines those procedures that transform a substance of concern into a different material. Regarding the similarity, transformation is the paper reusing plant that turns the message into a shoe box. The two main methods of transformation are physical (transformations triggered by physical regulations, such as dangerous decay) and chemical (transformations instigated by chemical or biological responses, such as disbanding)(Turner & Hurst, 2001).

1.5 Advection-Diffusion Equation

Essentially, transport happens in fluids over the progression of advection and diffusion. The concentration distribution of a pollutant released into the air may be described by the advection-diffusion equation which is a second-order partial differential equation of parabolic type. The mathematical equation for advection-diffusion equation is given as (Turner I & Zhuang, 2003):

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} = \frac{\partial}{\partial x}(K_x\frac{\partial c}{\partial x}) + \frac{\partial}{\partial y}(K_y(\frac{\partial c}{\partial y}) + \frac{\partial}{\partial z}(K_z\frac{\partial c}{\partial z}) + s \qquad (1.5.1)$$

where K_x, K_y, K_z are the eddy diffusivity along x, y and z direction respectively. The following assumptions are made in the solution of the equation (1.5.1):

- Steady state condition is considered (i.e. $\frac{\partial c}{\partial t} = 0$).
- The vertical velocity component (w) is neglected in comparison to horizontal velocity components (u and v).
- x-axis is oriented in the direction of mean wind (i.e. $u = U$, $v = 0$).
- Downwind diffusion is neglected in comparison to transport due to mean wind (i.e. $\mid U\frac{\partial c}{\partial x}\mid\gg\mid \frac{\partial}{\partial x}(K_x(\frac{\partial c}{\partial x}))\mid$).

Application of these assumptions in equation (1.5.1) leads to the steady state advection diffusion equation for dispersion of a non-reactive contaminant released from continuous source as (Seinfeld & Peterson, 1977):

$$U\frac{\partial c}{\partial x} = \frac{\partial}{\partial y}(K_y\frac{\partial c}{\partial}) + \frac{\partial}{\partial z}(K_z\frac{\partial c}{\partial z}) \qquad (1.5.2)$$

where x, y and z are coordinates in the along-wind, cross wind and vertical directions respectively. C is the mean concentration of pollutants and U is the mean wind speed in the downwind direction. K_y and K_z are eddy diffusivities of pollutants in the crosswind and vertical direction respectively.

Mathematical models are in 2 categories, namely: statistical and deterministic models. Statistical models are founded on the analysis of previous monitoring air quality information. Deterministic models are founded on the mathematical explanation of physical and chemical methods taking dwelling in the atmosphere. These models are constructed on mathematical equations and express energy, momentum and conservation laws of mass. The Advection-Diffusion Equation is a deterministic model (Arya, 1999).

Founded on the Advection-Diffusion Equation, atmospheric dispersion modelling according to (Holmes & Morawska, 2006) customs mathematical algorithms, to define the atmosphere, spreading and chemical and physical developments in the interior of the cloud, to explain how air (or water) chemicals diffuse in the ambient atmosphere (or water body) as well as to compute concentrations at various places. These representations are used to estimate and/or forecast the downwind absorption of air or water pollutants or contaminants discharged from causes such as manufacturing plants, vehicular circulation and water contaminations from accidental chemical releases. Although the models are merely estimation models, they estimate concentration discharged from any detail established of chemical, at any site, at any period of time and with high levels of confidences (Nagendra & Khare, 2002).

According to Seinfeld & Peterson (1977) the atmospheric diffusion algorithms existed for a long time to define the spreading of airborne pollutants within the tempestuous atmosphere. The use of analytical solutions of this equation was the first and remains the most convenient way for modelling air pollution problems. Air dispersion models are built on analytical explanations which have quite a few advantages compared to numerical models as all the persuading parameters are clearly articulated in a mathematically closed form. Analytical models are also beneficial in investigating the exactness and performance of numerical models.

1.6 Difference between diffusion and advection

Diffusion and advection both transport the pollutants from one place to another, in different ways. The common difference is:

- Advection spirits one way (downstream);
- Dispersion moves in all directions (irrespective of a stream direction).

Below is the mathematical formulation:

- Advection $\frac{u\partial c}{\partial x}$ has a first-order derivative, which means that if x is replaced by -x the term changes signs (anti-symmetry);
- Diffusion $D\frac{\partial^2 c}{\partial x^2}$ has a second-order derivative, which means that if x is substituted by -x the term does not alter sign (symmetry).

1.7 Air dispersion models

The above-mentioned models are separated into diverse sets of source physiognomies such as point, line and area sources or on the basis of geographical of the region as flat or complex ground. They can as well be categorized according to the dimensions of the field they are defining (Bluett et al., 2004): Short distance models (distance from source less than 30-50 km), Mesoscale models (concentration fields of the order of hundreds of kms) and Mainland or Universal Flow models.

Some of the models can, moreover, be grouped according to the time resolution of the concentration formed: occasional models (sequential determination of less than an hour), Short-time models (sequential determinations larger than or equal to an hour and less than or equal to 24h) and climatologically models (by means of determinations bigger than 24h, seasonal or yearly are the common ones).

The overview of a dispersion coefficient as a function of time in a given distance from a source might permit obtaining outcomes which are physically acceptable; then again, their dependence on time brands it demanding to give the equation of diffusion in a static coordinate system given that several sources have to be preserved concurrently. Notwithstanding the commonly used limits of the slope theory has been largely exercised since a) modest, b) it defines the diffusive passage in the context of an Eulerian, besides nearly all dimensions are Eulerian view, c) outline fallouts that correspond with the data that were obtained experimental as well as more multifaceted models, e) they require less input i.e. not expensive (Chung, 2010).

In addition to starting the basis for a widespread and active body of present research in atmospheric dispersion modelling, the results are used to demonstrate basic techniques from partial differential equations (Greens functions, Laplace transforms, a asymptotic, special functions), constrained optimization (linear least squares), numerical analysis, and inverse problems.

Basically, to achieve these, a computer program was used that explained the equations and algorithms of mathematics which describe the contaminant diffusion. Those models are imperative specifically to governmental representative that are responsible for handling and shielding the ambient air quality. The models also stream data to support in the designing of the best control policies to diminish emissions of detrimental air pollutants (Sutton, 1932).

1.8 The aspect of climatic change

The aspect of climate differences has a key part in defining the dispersion, distribution, direction as well as the conveyance of contaminants in the atmosphere. Consequently, it is really essential to study and consider the climatic conditions of an area the study is carried out.

1.9 Air quality standards and regulations

Importantly, the purpose of modelling air dispersion is to measure the negative effects of a certain facility or movement on the atmosphere. The influence is measured by means of linking the expected concentration of the contaminant at ground level to an orientation level. The ambient air quality standards and limits are the commonly generally used reference for contrast in the field of environmental fluid flow. The data obtained are compared to the World Bank Guidelines (Table 1.1) for stack emissions.

Table 1.1: Wolrd Bank Guidelines (http://www.his.com/ mesas/OccupHealth.htm).

Pollutant	Stack Emissions/ Concentrations
Particulate Matter	$50mg/Nm^3$
Nitrogen Oxides	Oil: $460mg/Nm^3(130ng/Jor225ppm)$
Sulfur Dioxide	Concentration not to exceed $2000mg/Nm^3$.

1.10 Statement of the problem

Knowledge of the relation between emissions by a source and pollutant concentrations in the air at later times and other places (that is, the source/receptor relationship) is essential to calculating the exposure of humans to these pollutants and hence to predicting the health impacts resulting from these source emissions. Mathematical models have evolved as the most practical means to relate source emissions to the subsequent air pollution concentrations. This thesis models the air emissions from diesel fuelled generated electric power plants located in Walvis Bay, Namibia. The output of the models are then used to determine the potential impacts of the air emissions on the surrounding environment.

1.11 Objectives of the study

The General objective of this study is to develop a mathematical model of emissions from diesel-fuelled energy sources (diesel-fuelled power plant) at Walvis Bay, Namibia.

Specific objectives are

1. To analytically model the relationship between diesel power plant energy output, emissions, and subsequent air quality impacts using the advection-diffusion-reaction equations.

2. To assess possible ground level air pollutants' (Sulfur Dioxide and Nitrogen Dioxide) concentrations and assess any negative environmental impact, taking the air quality standards as a measure of impact.

3. To apply the developed model to Namibia's climatic conditions for use as a decision support tool for energy and environmental planning in Namibia.

1.12 Proposed research design

The researcher first identified the diesel emissions sources from the Anixas power plant in Namibia. These emission sources were then represented by appropriate energy output and emission variables from which the model was developed. The emission dispersion model was then solved using appropriate techniques to obtain the potential environmental impacts of the emissions.

Data needed as input in the model were obtained from Namibia's meteorological conditions. MATLAB was employed in the model solution and simulation of the emissions and environmental impacts developed. The data needed for the power plant are:

- Energy and emissions data for the Anixas diesel power plant in Namibia.
- Local weather and topographic data for Walvis Bay, Namibia.
- Air quality standard for Namibia.

1.13 Significance/contribution

The ongoing research on the energy models in the Namibian energy system needs additional study. Each country has its own unique energy resources, economic demand patterns, and energy security concerns. Thus, the best strategy for each country will be different. The significance of this study is for a better sympathetic of the environmental health effects of the Anixas power plant.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Consumptions of energy in Namibia has been rising with 3.5% every year on average as well as the consumptions of electricity with 5.6% even though pre-maturation rates differ notably from one year to another. During the period of 2000-2010 the overall hike for energy consumptions has been 38% while for electricity consumptions stands at 79%. The electricity creation capacity rises very little at the same time. The increase throughout this time has been nearly powered by the energy which was imported in the form of petroleum products and electricity (NamPower, 2013).

The main supply of electricity in Namibia depends mostly on the electricity which is imported from outside the country (65% in 2011). The inter-connector capacity for electricity used country wide at present stands at 900 MW, of which 600 MW comes from South Africa and the 300 MW Caprivi Link in north-east linking the Namibian system to the Zambian grid, which comes with an option for 300 MW increase in capacity when Phase 2 of the project is realized. The rest of the electricity supply is generated in practice by the Ruacana 332 MW hydro power plant located on the northern boundary river between Namibia and Angola. The plant contains three 80 MW turbines and a fourth newly commissioned 92 MW unit. During 2014, the runners of three older turbines were replaced, increasing the capacity by 15 MW to a total of 347 MW.

There are also two diesel plants (Paratus and newly built Anixas) in the town of Walvis Bay (24 MW and 22 MW respectively) and a coal fired condensing plant, Van Eck, in the capital city, Windhoek. The coal plant has four units rated 40 MW each, but currently in practice the maximum electricity output is a fraction of the total capacity, about 50 MW. Van Eck power plant remains a peak power station. In addition to these, a small pilot plant utilising a widely available domestic biomass fuel, invader bush, is located about two hundred kilometres north of the capital. The capacity of the plant is 250 kW (NamPower, 2013).

Basic information on these power generation plants in Namibia can be found in table 2.1

Table 2.1: Power Plants located in Namibia (Ministry of Energy, 2010).

Power plant	Type	Build	Capacity
Ruacana	Hydro power station	1978	347 MW
Paratus	Diesel power station	1976	24 MW
Anixas	Diesel power station	2011	22 MW
Van Eck	Coal-fired power plant	1973	120 MW
CBEND	Small scale biomass plant	2010	0.25 MW
Tsumkwe	PV - diesel hybrid	2011	200kWp PV +300kW

2.2 Fluids in the environment

Types of substances are grouped as they appear in one of two states namely, fluid or solid. Substances that have a rigid shape that can be modified as a consequence of stresses are know as solid elements. This shape modification is described as deformation or strain. Various types of solids are identified by different relationships between the shear stress and the strain. A state of equilibrium with the stresses applied on that body is described as strained solid. When applied stresses vanishes, the solid body relaxes to its original shape.

The shape of a fluid body is determined by interfaces and solid boundaries with other fluids. Not like in solids, even an infinitesimal shear force changes the appearance of fluid elements. Differences between various forms of fluid are identified by different relationships between the shear stress and the rate of strain. When applied stresses vanish, fluid elements do not return to their original shape. In addition, fluids normally do not support tensile stresses, even though in many scenarios they strongly resist normal compressive stresses. Mostly, they can be seen as incompressible materials or materials subject to incompressible flow, that is, their density is not a function of pressure. Generally, fluids may be divided into liquids, for which compressibility is generally not considered, and gases, which are compressible fluids. The volume of a liquid mass is almost constant, and it takes the smallest portion of a container in which it is contained. It also has a horizontal free surface in a stationary container. A gas always expands and takes the whole volume of any container. Hence, gases like air are usually well described in the atmosphere using incompressible flow theory (Carslaw & Jaeger, 1959).

All forms of life on earth are immersed in a fluid or another, either the air of the atmosphere or the water of a river, lake or ocean; even, soils are permeated with moisture. So, it is no exaggeration to say that life, including our own, is bathed in fluids. Our industrial systems, which release pollution on a continuing basis, would not be permissible if transport and dilution of nearly all emissions by ambient motions of air and water is not present (Cooper & Alley, 2002).

In sum, natural fluid motions in the environment are vital. We have a strong incentive to study the naturally occurring fluid flows, particularly those of air in the atmosphere and of water in all its streams, from underground aquifers to surface flows in rivers, lakes, estuaries and oceans.

The study of these flows has received considerable attention over the years and has spawned several distinct disciplines: meteorology, climatology, hydrology, hydraulics and oceanography. Whereas, the particular objectives of each of these disciplines, such as weather forecasting in meteorology and design of water-resource projects in hydraulics, encourage disciplinary segregation, environmental concerns compel experts in those disciplines to consider problems that are essentially similar. These includes, the effect of turbulence on the dispersion of a dilute substance, the transfer of matter or momentum across an interface, flow in complex geometries, the rise of a buoyant plume, and the impact of flow over a biotic system (Cooper & Alley, 2002).

2.2.1 Assumptions in Continuum

Altogether resources are composed of distinct molecules subject to relative movements. However, in the context of fluid mechanics we contemplate the fluid as a continuum. We are commonly concerned in the macroscopic manners of fluid substances, such that the lowest fluid mass of concern generally is made up of a fluid particle that is much greater than the mean free path of a sole molecule. Hence, it is likely to disregard the discrete molecular structure of the matter and to define it as a continuum. The continuum method is lawful if the distinctive size, or length of the flow system (e.g., the diameter of a solid sphere immersed in a flowing fluid) is much greater than the mean free path of the molecules. For example, in a normal atmosphere the molecular free path is of the order of 10^{-8} m, but in the high altitudes of the atmosphere the molecule mean free path is of the order of 1 m. Consequently, for us to analyse the dynamics of a complex gas in such heights a kinetic theory method would be essential, relatively than the continuum method (Seinfeld & Pandis, 1997).

The study of environmental flows is also fully integrated in the contemporary emphasis on environmental impacts and sustainable life on planet Earth. According to physicists, the world scientific community will be occupied during the 21st century in large part by problems related to the environment, particularly those stemming from the concern over climate change (Rodhe et al., 2000) as well as many other problems spanning a wide range of spatial and temporal scales. This marks the first time in the history of science that environmental problems lie at the forefront of scientific research.

2.2.2 Partial Differential Equations

All basic processes characteristic of environmental fluid mechanics can be analysed as partial differential equations which is commonly abbreviated as PDEs. Partial differential equations are a result of a functions for which results are required (e.g., concentrations, velocities, temperature, etc.) or be likely to rely on one or more spatial coordinates as well as time. Utmost, the equations of concern comprise diffusion processes, which include second-order spatial derivatives. The result of the suitable differential equation(s) subject to suitable initial and boundary conditions gives the foundation for mathematical model of the physical problem. In the following paragraphs, we analysed the simple forms of partial differential equations faced with environmental fluid mechanics problems (Lawrence, 1998).

Credentials of the partial differential equation linked with the specific problem of concern are of most significant. Diverse measures of convergence and stability are typical of each type of partial differential equation, as given below. The equation offers the elementary instruction for the establishment of a mathematical problem that can be used to the solution of that real world problem. In cases of numerical models, specific procedures for the formulation of the numerical system are used for the specific differential equation that is linked with a specified problem. Problems of environmental fluid mechanics can be categorized into two universal classes: problems of equilibrium and problems of propagation (Lawrence, 1998).

The universal set up of a second-order linear Partial Differential Equation in a two dimensional is given by

$$a\frac{\partial^2\varphi}{\partial x^2}+b\frac{\partial^2\varphi}{\partial x\partial y}+c\frac{\partial^2\varphi}{\partial y^2}=f \tag{2.2.1}$$

such that a, b, and c are constant coefficients. A linear combination of coefficients multiplied by small order derivatives of the dependent variable φ is represented by f.

The technique and procedure of the solution of a Partial Differential Equations subject to initial and boundary conditions relies on the type of the Partial Differential Equation. It is well known to categorise Partial Differential Equations based on the relationships between the coefficients of equation (2.2.1):

If

$$b^2-4ac>0 \tag{2.2.2}$$

then the Partial Differential Equation is *hyperbolic*.

If

$$b^2-4ac=0 \tag{2.2.3}$$

then the Partial Differential Equation is *parabolic*.

If

$$b^2-4ac<0 \tag{2.2.4}$$

then the Partial Differential Equation is *elliptic*.

Hyperbolic, parabolic or elliptic is used to categorize partial differential equations branches from the analogy between the system of the discriminant (b^2-4ac) for partial differential equations and the system of the discriminant that categorises conic sections. If the Partial Differential Equation defines a domain with n -dimensions, then the characteristics, if actual characteristics exist, are surfaces of $(n-1)$ dimensions, along which signals, or evidence, propagate. If no actual characteristics exist, then there are no preferred paths of information propagation. Consequently, the presence or absence of characteristics has an important influence on the results of the partial differential equation.

First-order partial differential equations defines convection or advection of a property φ, given as a solute heat or concentration. The common formula of such an algorithm in the (x,t) domain, such that x defines a spatial coordinate and t defines a time, defined as

$$\frac{\partial\varphi}{\partial t}+u\frac{\partial\varphi}{\partial x}=0 \tag{2.2.5}$$

such that u is the advection velocity. If φ defines dissolved mass of a solute, then the second term in the above equation incorporates the process of solute mass being carried (advected) by a fluid particle as it moves through the domain. The location of any fluid particle is compared to its velocity u by a simple relationship representing the differential equation of the particle path line:

$$\frac{dx}{dt} = u \tag{2.2.6}$$

Hence, the path line of a fluid particle is defined as

$$x = x_0 + \int_{t_0}^{t} u dt \tag{2.2.7}$$

Beside, the path line of the fluid particle of the advection equation is stated as

$$\frac{\partial \varphi}{\partial t} + u\frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial t} + \frac{dx}{dt}\frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial t} = 0 \tag{2.2.8}$$

The last part of equation (2.2.8) indicates that φ is constant along the path line of the fluid element. This path line is the distinctive path related to the equation of advection. The first-order differential equation of the form given by Equation (2.2.5) is termed a first-order hyperbolic partial differential equation, and it has a single family of characteristic curves, along which the information propagates in the domain. A single first-order partial differential equation is always hyperbolic. In second-order hyperbolic partial differential equations there are two families of characteristic curves, along which the information propagates.

Parabolic and hyperbolic differential equations are typical of propagation problems. The propagation is in time and space. This means that parabolic and hyperbolic differential equations usually defines the problems of a property propagating in the domain. The features of the propagation of the property in cases of parabolic differential equations are different from those of hyperbolic differential equations. Elliptic partial differential equations generally concern equilibrium problems, i.e., ones that do not involve time derivatives.

A typical parabolic equation associated with environmental fluid mechanics is the equation of diffusion. In the (x, t) domain, the form of this equation is stated as follows

$$\frac{\partial \varphi}{\partial t} = \alpha\frac{\partial^2 \varphi}{\partial x^2} \tag{2.2.9}$$

where α is the coefficient of diffusion, or diffusivity. In numerous claims, an advective term is added, making an advectiondiffusion equation

The wave equation is a most hyperbolic equation connected with environmental fluid mechanics. Considering the domain (x, t), equation (2.2.9) is stated as

$$\frac{\partial^2 \varphi}{\partial t^2} = \alpha^2\frac{\partial^2 \varphi}{\partial x^2} \tag{2.2.10}$$

such that c is the propagation speed of the wave.

Laplace equation is a known elliptic equation, linked with environmental fluid mechanics. Considering the domain (x, y), equation (2.2.9) is stated as

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \tag{2.2.11}$$

The solution of a parabolic or hyperbolic partial differential equation, of the types stated by equations (2.2.9) and (2.2.10), can be attained, as long as adequate initial as well as boundary conditions are stated. An initial condition defines the values of the unknown variables and possibly their space derivatives at a time of reference. Boundary conditions define the values of the unknown variables and their space derivatives at the boundaries or other specific locations of the given domain. The solution of an elliptic partial differential equation of the type stated by equation (2.2.11) can be attained, as long as adequate boundary conditions of the domain are indicated. For elliptic partial differential equations there are no initial conditions, since time derivatives are not included.

To the solution of partial differential equations, there are three types of linear boundary conditions that can be applied:

All values of the dependent variable, φ, are specified on the boundaries of the domain:

$$\varphi = f(x, y) \tag{2.2.12}$$

such that

$$(x, y) \in G \tag{2.2.13}$$

such that G is defined as the surface of the domain. Boundary conditions of this type are defined as Dirichlet boundary conditions.

All values of the gradient of the dependent variable, φ, are specified on the boundaries of the domain:

$$\frac{\partial \varphi}{\partial n} = f(x, y) \tag{2.2.14}$$

given that

$$f(x, y) \in G \tag{2.2.15}$$

such that n represents a coordinate normal to the boundary G. Boundary conditions of this type are defined as Neumann boundary conditions.

A general linear combination of Dirichlet and Neumann boundary conditions:

$$a\varphi + b\frac{\partial \varphi}{\partial n} = c \tag{2.2.16}$$

such that a, b, and c are functions of (x, y). This type of boundary condition can be used to specify total flux, as will be described in later chapters.

It should be noted that besides linear boundary conditions, the domain may be subject to non-linear boundary conditions. Application of such conditions is generally very complicated.

2.3 The modelling theory of diffusion in the atmosphere

Modelling of dispersion customs mathematical algarithms relating the chemical, atmospheric, dispersion and physical developments prompting a contaminant produced from well spring outcomes of a specified geometry to determine the amount of emission at several receptors as a consequence of the release of pollutants (Holmes & Morawska, 2006).

Molecular diffusion is one the simple example of contaminants dispersion in the atmosphere, once matters travel from an area of great concentration to an area of little concentration. Nevertheless, other than the diffusion of molecular, plumes disperse because of additional multifaceted methods. These methods are mechanically as well as thermally created by turbulence and wind variations (Cooper & Alley, 2002).

2.4 Historical perspective of mathematical models.

Major components of civil engineering works have been fluid mechanics and hydraulics and were perhaps initially connected with problems of water supply in ancient civilizations. One of the first well-documented hydraulic engineers was Archimedes. His finding of the plain principles of buoyancy works nowadays as one of the essential building blocks in defining fluid behaviour. He also considered modest pumps for agricultural applications, and some of his plans are quiet in use currently. Additional primary engineers had to deal with moving water over large distances from sources to cities, as with the Roman aqueducts found in many parts of Europe and the Middle East (Holmes & Morawska, 2006).

These strategies required to include basic features of open channel flow, such as discovery the proper slope to attain a preferred flow rate. Remains of water storage and conveyance systems have also been found from some of the earliest civilizations known, both in the Near East and in the Far East. Rouse (1957) offers a fascinating history of the science and engineering of hydraulics, which is also shortened by Graf (1971), mostly as it tells to open channel flow. In a sense, these were the first kinds of problems that can be related with the field of environmental fluid mechanics.

To come up with the procedures for disposing of waste water was an important task for the early engineers. The humblest means of achieving this, which was in use up to the relatively recent past, comprised systems of gutters and drainage ditches, frequently with direct discharge into ponds or streams. Septic tanks, with associated leeching fields, are another instance of a simple waste water treatment scheme, and yet these can handle only relatively slight flow rates. Inside the last century, the practice of waste water collection and treatment has grew significantly, to allow fluctuating degrees of treatment of a waste stream before it is discharged back into the natural environment (Tyler, 1986).

This growth has been motivated by increased demands (both in quantity and in quality) for treating municipal sewage, as well as increased necessities for treating industrial wastes. Sanitary engineering, within the overall profession of civil engineering, usually allocated with designing water and waste water collection and treatment schemes. This has changed into the fashionable field of environmental engineering, which now incorporates the universal area of water quality modeling, for both surface and groundwater structures. This has imposed the incorporation of other fields of science, such as chemistry and biology, to address the wider range of problems now being faced in treating waste streams with a diversity of features and requirements (Tyler, 1986).

In addition to treating municipal or industrial waste water, environmental engineers presently are involved in answering problems of chemical fate and transport in natural environmental schemes, including subsurface (groundwater) and surface waters, sediment transport, and atmospheric structures. An acquaintance as well as considerate of fluid flow and transport practices is essential to define the transport and dispersion of pollutants in the environment, and chemical and biological practices must be incorporated to define source and sink terms for contaminants of concern. Distinctive types of difficulties might involve computing the projected chemical contaminant concentration at a water supply intake due to an upstream spill, evaluating the spreading of waste heat discharged from power plant condensers, forecasting lake or reservoir stratification and associated effects on nutrient and dissolved oxygen distributions, determining the relative importance of contaminated sediments as a continuing source of pollutants to a river or lake system, calculating the expected recovery time of a lake when contaminant loading is discontinued, or evaluating the effectiveness of diverse remediation options for a contaminated groundwater source. All of these types of difficulties need an thoughtful of fluid flow phenomena and of biochemical behaviour of resources in the environment (Tyler, 1986).

The most important aspect fuelling the development of mathematical air quality models has been the Clean Air Act (American Meteorological Society 1981). Models are put in place so that they validate agreement with governing norms as well as control supervisory bodies in the direction of thinkable emission control strategies for refining the quality of airborne (Hadlock, 1998). Through the inspiration of the Clean Air Act, air quality has been considered specifically to foresee the concentrations of contaminants like carbon monoxide, nitrogen dioxide, and ozone through the structure of the federal government for various years, excluding those of countless trace of toxic pollutants which have been previously of rising concern to wholesome results of investigation of which are expected to be a topic and order to the forthcoming (Pasquill & Smith, 1983).

In the beginning of the 1970s, coherent models have been advanced to the point that it was a potential to forecast the concentrations of emissions like Carbon dioxide has been mostly given by transport but not by the reaction of chemical in the atmosphere. After that, it was necessary to integrate the chemical in the atmosphere into the model to designate the active of contaminants, such as O_3 and NO_2 that are chemically vigorous upon the atmosphere. Photochemical air shed models had been developed at the beginning of 1980s, that could precisely predict the concentrations of NO_2 and O_3 as a function of emissions. Currently, the main problem in our skilfulness to designate the dynamics of those given air emission in town is the usefulness of high-quality input data, but not the model itself (de Nevers, 2000).

The previous 10 years saw rapid progress of statistical and empirical models of air quality. Various designs of the 1970s expected that the basin wide air standard altered in equal amounts in total, basin wide discharges (Hadlock, 1998). This override copy was added to basin wide release that was envisaged. Overturned duplicates were partial on request based on the overlook significant result facted resources variations and ambiance chemistry (Tyler, 1986).

2.5 Theory of turbulence

Turbulence is the term used to characterize the complex, seemingly random motions that continually result from instabilities in fluid flows. Turbulence is ubiquitous in natural fluid flows because of the large scales that these flows typically occupy. Transportation of particles of pollutants in the air from one site to the other is carried out by means of turbulence. The combined random motion involving a group of various molecules is known as Turbulence (Turner & Hurst, 2001) and is made up of both mechanical and thermal eddies. Eddies are macroscopic random fluctuations from the average movement. These turbulent eddies are accountable for the dispersion of contaminants in the atmosphere. Eddies scatter pollutants by interrupting the plume, replacing a set of concentrated contaminants in a plume with a set of clean air from a distance away from the plume, subsequently weakening the plume and spreading it in both vertical and lateral directions (Cooper & Alley, 2002).

Turbulent from mechanical is formed over the collaboration among the forces in the horizontal direction applied by one layer on an adjacent layer together with the slope of the mean velocity with advancement. The more the mechanical turbulence, the greater the roughness elements or the tougher the wind (Antonio et al., 2004). Therefore, the irregular surfaces such as forests yield more eddies compared to a smooth surface such as ice, Buildings plus other obstacles raises mechanical turbulence since these obstacles rise the horizontal forces that slow down the mean wind (Sutton, 1932).

The ground absorbs thermal energy coming from the sun. The absorbed heat is transported into the lower atmosphere by means of convection or conduction; consequently creating thermal eddies. When there is strong insulation than the energy from the sun is feeble extra eddies will be produced (Briggs, 1975).

2.5.1 Spalart-Allmaras one-equation model

Spalart-allmaras one-equation model solves a single conservation equation of a Partial Differential Equation (PDE) for the turbulent viscosity. This conservation equation contains convective and diffusive transport terms, as well as expressions for the production and dissipation of kinematic turbulent viscosity.

2.5.2 Developement of the model

The central idea which lies at the heart of development of the Spalart Allmaras model is the fact that evolution of any scalar quantity, like kinematic turbulent viscosity, subject to the conservation laws is given by the following differential equation:

$$\frac{\partial v_t}{\partial t} + \nabla.\phi(x,t) = s(x,t) \tag{2.5.1}$$

where $\phi(x,t)$ is the flux of v_t and $s(x,t)$ is the source or sink term which combines the effects that create or destroy v_t.

Now, the flux $\phi(x,t)$, i.e. the rate of transport of v_t, takes into consideration the changes in v_t due to its convection $\phi_C(x,t)$ by the mean velocity field U and diffusion $\phi_D(x,t)$ due to its uneven distribution. The convection flux can be rewritten as

$$\phi_C(x,t) = Uv_t \tag{2.5.2}$$

As a result, the transport equation for v_t takes the following form:

$$\frac{\partial v_t}{\partial t} + \nabla.(Uv_t) + \nabla.\phi_D = Production \quad - \quad Destruction \tag{2.5.3}$$

In tensor notation, the above equation can be written as

$$\frac{Dv_t}{Dt} \equiv \frac{\partial v_t}{\partial t} + U_j \frac{\partial v_t}{\partial x_j} = Production \quad + \quad Diffusion - Destruction \tag{2.5.4}$$

where Diffusion $= -\nabla.\phi_D$

Most of the terms in the RHS are formed mainly by drawing an analogy of v_t with the turbulent energy. The basic premise for drawing this analogy is that v_t may be regarded as the ability of the turbulent flow to transport momentum and hence it can be directly related with the turbulent energy.

2.6 Concentration determination

Tripathy & Panigrahi (2002) studied the effect of concentration on the distance from the point source emission. They concluded that the full concentration is continuously noted nearby the source. This was because of very slight wind velocity taken and also on solar insulation influences. The spread is very noticeable within the radius of 1 km as well as rapid fall in the concentration from the source is eminent in all the seasons after 1.6 km and 5 km.

The witnessed and projected ground level SO_2 concentrations and critical distances are compared to generalise the conventional Gaussian plume dispersion formulas for the non-stationary circumstances of the inversion fragmentation at large power plants (Okamoto et al., 2001). The expected full ground level concentrations and related distances downwind with field records collected during an extensive survey conducted was compared and recommendation was done (Wang, 1977).

An analytical model for air contamination passage and deposition from a point source was compared. The model is attained from the analytic explanation of the atmospheric diffusion equation with the coefficient of eddy diffusion engaged to be functions of down-wind distance and the average wind velocity taken to be constant. The model results for the classes of deposition phenomena were discussed. The value of these velocities was revealed to have major effects upon the ground level contaminant concentration, the vertical plume shape and the pollutant deposition flux in the ground (Turner & Hurst, 1977).

A new model for plume rise from multiple sources built on simulated stack theory was developed. The comparison made in the paper ofAnfossi et al. (1978) suggests that virtual stack concept is consistent with experimental data and has more general application and for short term pollution concentration detailed input parameters are required.

The cyclical changes of SPM from thermal station within a distance of 5 km using a Gaussian plume model was studied by (Goyal & Sidhartha, 2004). The research was done by means of both monitoring and modelling. Tentative measurements were found from a monitoring network within the power station. The calculated values of particulate matter at six receptors, within the thermal power plant were used to evaluate the seasonal influence of the thermal power plant on air quality. The meteorological data on wind speed, wind direction, cloud cover and surface temperature for two years were achieved from India Meteorological Department (IMD). The mixing height was obtained from National Physical Laboratory, Delhi, through Radio sound method.

The control valuation and justification policy for air quality transformation due to a proposed thermal power plant in the Mangrol area of Chittorgrd district of Rajasthan was studied in detail by (Mohan & Kandya, 2009). This study includes sources of emission from tall stacks and also fugitative emission in the premises from wind erosion, load in onto piles, load out from piles and vehicular traffic. The impact of pollutants has been assessed in terms of both short term (i.e. 8 hourly averages) as well as long term concentrations. ISCTS3 model has been used for short term concentration predictions and CDM (Climatological Dispersion Model) for long term concentration predictions and fugitative emissions were estimated using PEM (pollution Episodic model). It was found that the contribution due to the proposed plant was very low (Jeong et al., 2005).

The overall mathematical organisation of a distinctive numerous source plume model for estimating concentrations of inert air pollutants in a city was scrutinised. It was found that once the emissions and climatic situations are anticipated to be statistically self-determining or uncorrelated. then the short term manifold source formulation tips to extensive term average concentration values equal with those assumed by meteorological model.

An operational model for the diffusion of gases from manifold sources in an urban area was obtained by (Turner & Hurst, 2001). A diffusion equation adapted to use area as an alternative to a point sources was used with a basis list of SO_2 emission to compute 24 hour concentrations (0 to 12 $\mu g/m^3$) at 1 mile breaks. Wind velocity and stability were averaged by 2 hour periods to assess the diffusion equation. Calculated concentrations exceeded the observed concentrations more frequently (28%) than observed concentrations (14%) for 32 stations for 35 periods. As soon as zero values of both considered and detected, concentrations were omitted, 70% of the intended values was inside a factor of two of the detected values. Under calculations upwind of main sources was perhaps because considered concentrations at the centre of a square mile area established no influence from bases within that part, but only from other parts. Outcomes showed that 24 hour concentrations and their degree in urban areas may be assessed for a variety of climatic circumstances using the source inventory diffusion method (Lin & Hildemann, 1996).

The expectations of simple Gaussian plume model for NO_x concentration at ground level using a continuous monitors over a six month period and upward at three elevations within the plume using a particularly built bag analyst was compared by (Roy & McCartney, 1980). Values of wind speed and direction at 5 m and solar irradiance and air temperature at screen height were noted at 2 minute breaks throughout the measurement period. Stability was classified based on Turner method. The measured concentration values have been associated with the calculations of the simple Gaussian plume model. It institutes that the ground level concentrations at 1 to 2 km distance from the source could be projected consistently. The predicted mean NO_x concentration was between $20\mu g/m^3$. The measured mathematics mean NO_x concentration for the same period was $27\mu g/m^3$. The results of the comparisons between the calculated and measured concentration for the ground level measurements, both short term averages and long term averages, indicated that the model could foresee ground level concentrations within a factor of two (Okamoto et al., 2001).

2.7 Source modelling

In order to model pollutant dispersion as correctly as possible, the source must be well-defined. Sources for air contamination modelling can be broken up in four types. The utmost exact symbol of the real-life situation is selected and fully detailed by the user. Multiple sources of similar or diverse types are frequently used (Arya, 1999).

Fisher & Macqueen (1981) have considered the importance of source treatment on mathematical outcomes together with wind tunnel measurements. They resolved that both type and strength of source have an effect on results and that clear source modelling strategies should be existing to modellers. In case that the characteristics of sources are not known and measurements are impossible, both worst- and best- situations should be modelled.

The available different sources in dispersion models are outlined below and presented in figure 2.1:

> Point source: this source is used for releases from small openings including industrial stacks. Several variables need to be clearly explained in order to use it correctly.
>
> Area source: frequently is used for sources with great surface areas of uneven forms.
>
> Line source: in most cases is used for roads, thin buildings as well as airport. Pollutants are released from a simulated line in the modelling domain.
>
> Volumetric source: By the combination of area sources in both vertical and horizontal directions. Pollutant releases from within buildings normally use volumetric source.

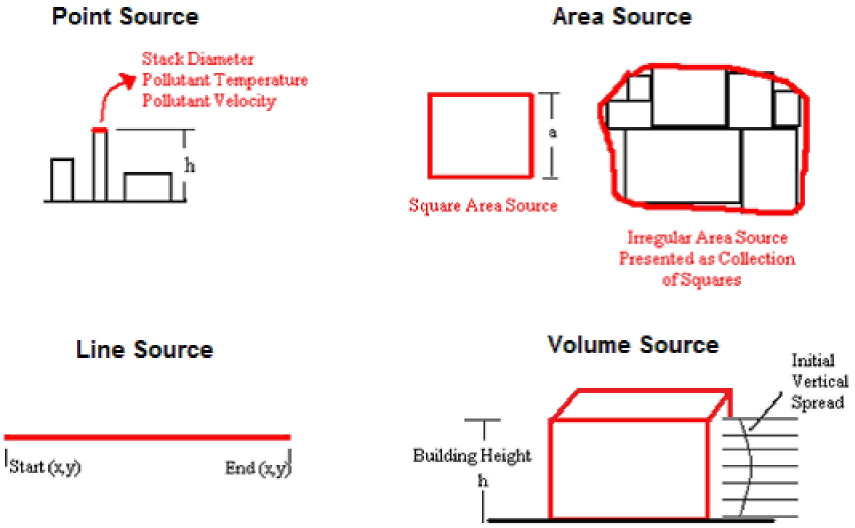


Figure 2.1: Source types used by dispersion models (Bluett et al., 2004)

The amount of emission is also a significant factor for properly modelling of point source. It is not suitable for the emission rate to be constant. In most cases, contaminants are not released at a continuous rate from the point source. The rates of emission normally differ with time: for instance, the contaminant emitted from a process that does not function 24 hours a day or 7 days a week is not continuous. Alternative instance is the variable emissions due to the rate of process within working hours (e.g. rate of production). Pollutant emission rates can also be affected by season and temperature, with every pollutant category behaving otherwise according to its properties (Mohan & Kandya, 2009).

The nature of the source could also have an impact on the variable emission rate. Great area sources often produce irregular pollutant releases over the surface and in some cases even because of the wind size over that surface. For deliberation of such source behaviour, unsteady inlet environments and source emissions are taken into account (Mohan & Kandya, 2009).

2.8 Model categories

Mathematical algorithms have been used for the modelling of dispersion of contaminant in the atmosphere. Examples of few that employ mathematical approaches, includes (Bluett et al., 2004):

- Box approach
- Gaussian approach
- Lagrangian approach
- Eulerian approach

The nature of the pollutant in use, the source where the pollutant is coming from and if they come in a form plume or puff method determine the model to be used.

2.8.1 Box approach

The simplest of all modelling systems is the box model and is founded on the mass of conservation. Airshed is preserved as a box into which pollutants are discharged and where they undergo chemical and physical developments. We assume that the concentration of air present in the box is similar. The model customs that assumption to estimate the average pollutant concentration anywhere within the airshed (Tyler, 1986). The mathematical represents the Box Model uses mathematical representation as given below:

$$\frac{dcv}{dt} = QA + uC_{in}WH - uWH \tag{2.8.1}$$

given that

- Q = the rate of emission
- C = concentration of pollutant
- V = capacity (volume)
- C_{in} = species concentration toward the inside of the airshed
- A = area of the box in a horizontal (L x W)
- L = length
- W = width
- u = wind speed
- H = mixing height

Although the Box model is important, modelling of air emission concentration is inappropriate due to the fact that it takes the development of pollutants inside the box model in the absence of providing slightly data local concentrations of the contaminants (Holmes & Morawska, 2006).

2.8.2 Lagrangian approach

In comparison to Box models, Lagrangian models outline an airshed as a box covering a preliminary chemicals concentration. Though, the Lagrangian model then trails the route of the box as it moves downwind. The Lagrangian model then computes the air pollution dispersion by calculating the statistics of the routes of a large number of the pollution plume packages. The Lagrangian model practices a moving frame of orientation. The mathematical algorithms for Lagrangian is given as (Nagendra & Khare, 2002):

$$\langle c(r,t)\rangle = \int_{-\infty}^{t} \int p(r,t \mid r',t')s(r',t')dr'dt' \tag{2.8.2}$$

such that

$\langle c(r,t)\rangle$ = Average pollutant concentration at location r at time t.

$p(r, t \mid r', t')$ = the probability function that an air parcel is moving from location r at time t to location r at time t.

$s(r', t')$ = emission source term

The Lagrangian model foresees the concentration of pollutants that are built upon a moving reference network (Chung, 2010). Its weakness is that they are restricted when outcomes from its estimate are associated with real quantities, because quantities are completed at motionless points.

2.8.3 Eulerian approach

The similarity between Lagrangian approach and Eulerian approach is that its paths the movement of a huge number of pollution plume packages as they change from their original location. Eulerian approach utilizes a static 3-D Cartesian coordinate as a way of evidence relatively than a motion frame of reference. If the pollutant is known, the Eulerian model uses mathematical equations given by conservation of mass. Mathematical representations is normally as (Stockie & Lushi, 2010):

$$\frac{\partial \langle c_i \rangle}{\partial t} = -\bar{V}.\nabla \langle c_i \rangle - \nabla.\langle c_i' V' \rangle + D\nabla^2 \langle c_i' \rangle + \langle S_i \rangle \tag{2.8.3}$$

given that

$V = V' + \bar{V}$

V = vector $V(x, y, z)$

$\bar{V}$ = wind field vector average

V' = changing wind fields vector

$c = \langle c \rangle + c'$

c = chemical concentration

$\langle c \rangle$ = total chemical concentration; $\langle \rangle$ denotes average

c'= fluctuating chemical concentration

D = molecular diffusive

S_i = source term

From the mathematical point of view, $-\bar{V}.\nabla \langle c_i \rangle$ is known as hyperbolic, the turbulent dispersion is known as parabolic and the set of differential equations defines the source term which make it difficult to solve. Hence it can be computationally expensive to solve (Settles, 2006).

2.8.4 Gaussian plume approach

Gaussian model is the greatest known model for the dispersion of atmospheric pollutant in mathematical modelling. In statistics, Gaussian is defined as a group of arranged values that trail a bell-shaped curve distribution (Chung, 2010). This type of model accepts that the pollutant disperses according to the regular statistical distribution (Holmes & Morawska, 2006). At the point of release, the pollutant concentration is at an extreme and declines in both lateral and vertical directions succeeding the normal distribution. The Gaussian equation is used for emission from a point source overall. Its mathematical algorithms is given as (Cooper & Alley, 2002):

$$c = \frac{Q}{2\pi u \sigma_y \sigma_z} e^{\frac{-y^2}{2\sigma_y^2}} e^{\frac{-(z-H)^2}{2\sigma_y^2}} \tag{2.8.4}$$

such that

- C - is a concentration at a given points (x, y, z), micrograms per metre cube.
- Q - is emission rate for specified pollutant, micrograms per seconds.
- u - is a wind speed, release at given height
- σ_y and σ_z is a dispersion coefficients also know as standard deviation
- y - is horizontal distance from the source.
- H - is effective stack height ($H = h + \Delta h$) where h = physical stack height and Δh= plume rise
- z - is vertical distance.

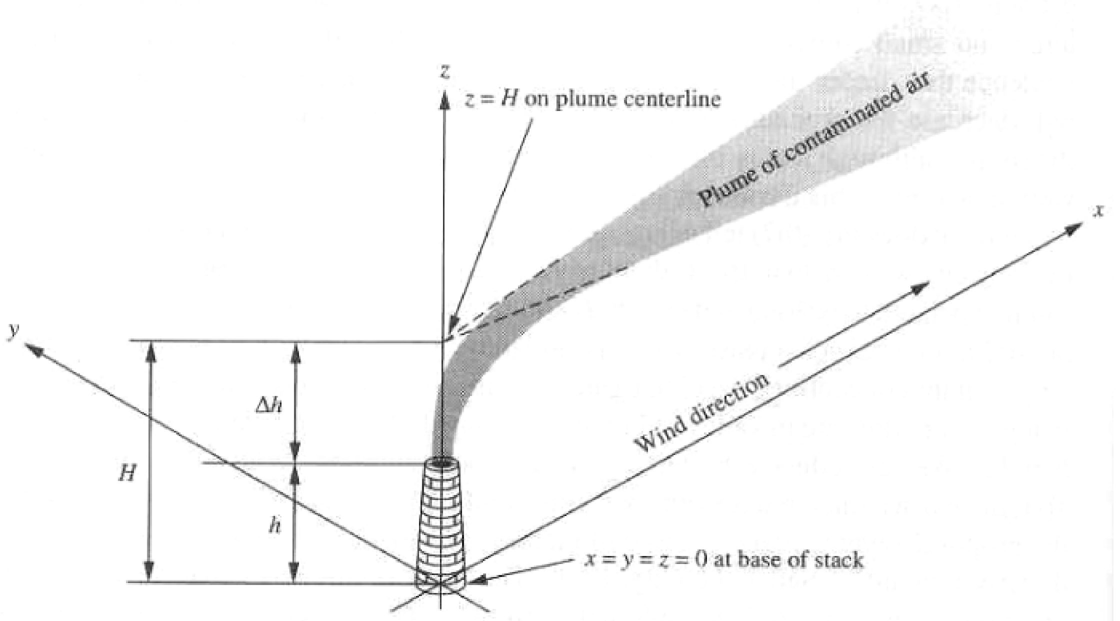


Figure 2.2: Coordinate system for Gaussian Plume Idea (Ahmed & Hossain, 2008).

From the figure, the source of the coordinate system is located at the base of the smokestack with the x axis allied in the downwind direction. The contaminated gas stream or plume increasing from the smoke stack (Δh) and then travel in x direction and spread in the y and z direction as it travels. The plumes generally increase higher above the smoke stack since they are released at higher temperature than atmosphere and with vertical velocity. For the calculations, the plume is assumed to be discharged at coordinate; $(0, 0, H)$ where H is effective stack height which are the summation of the physical stack height and plume rise ($h + \Delta h$). The smoke emitted at the point source is assumed to be a non-buoyant pollutant at emission rate Q (g/s) and setbacks in x direction with velocity u that is self-governing of time, elevation or location(Crank, 1975).

If the molecular diffusion alone was producing the plume to mix with the neighbouring air, the plume will spread gradually as a thin streak moving straight down the sky.

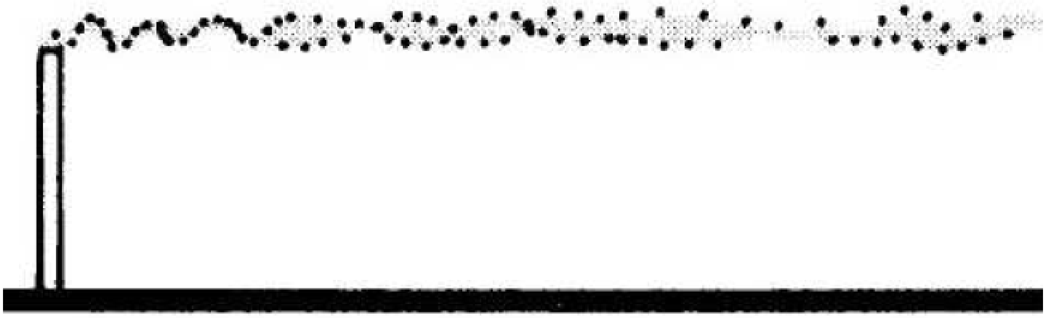


Figure 2.3: Time exposure of a visible plume (Arya, 1999).

The spread of the plume does not depend on molecular diffusion alone. The key cause of the spreading is the large scale turbulent mixing that happens in the atmosphere. The twisting behaviour is caused by the turbulent motion of the atmosphere that is superimposed on the plume's large scale linear motion instigated by horizontal wind (Erwin, 1989). This turbulent motion is random in nature; a snapshot taken after a few minutes after the first will show a diverse snapshot in different places. Yet, time averages the variation and consequently the concentration will seem a bit unchanging and symmetrical (Arya, 1999).

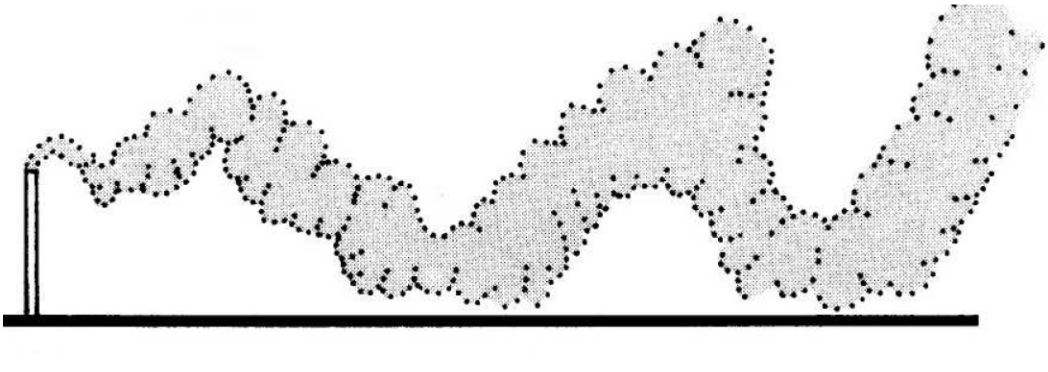


Figure 2.4: Snapshot of a visible plume (Arya, 1999).

The Gaussian model is based on the following assumptions (Stockie, 2011):

The emission rate released should remain constant and uniform.

Speed and wind direction remain the same.

Diffusion towards downwind is insignificant in comparison to diffusion from vertical and crosswind.

The ground is taken to be relatively flat.

There is no deposition or absorption of the pollutant.

Diffusion of the pollutant trails a Gaussian distribution.

Application of Gaussian plume approach to the particles of dispersion in the modelling, it has limitations. These limitations are results of the use of steady state approximations without taking into account the time needed for the pollutant to travel to the receptor and the vertical particle movement due to gravity throughout this time. Progressive Gaussian models, recently, have been established that can solve most of the limitations problems in Gaussian models that was developed earlier. AERMOD and ADMS are the new generation models developed with progressive algorithms to overwhelmed the early Gaussian model limitations (Carslaw & Jaeger, 1959).

2.9 The atmospheric stability

The propensity of the atmosphere to upsurge or overpower vertical movement moves the concentration of pollutants. A stable atmosphere will cause complex pollutant concentrations and prevent dispersion of pollutants in the atmosphere. An unstable atmosphere, pollutant concentration will be fewer and dispersion will upsurge (Swart, 2016)
Pasquill categorized the atmospheric turbulence into six stability classes, namely, A, B, C, D, E and F. Stability class F is the most stable or described as the least turbulent class. Solar radiation increases atmospheric instability through warming of the earth surface so that warm air is below cooler air causing vertical mixing. Clear nights push conditions toward stable as the ground cools faster establishing more stable conditions and inversions. Wind increases vertical mixing, breaking down any type of stratification and pushing the stability class towards neutral.

Other parameters that can define the stability class of the atmosphere are

Temperature gradient

fluctuations in wind direction

Richardson number - which is a dimensionless number that expresses the ratio of the buoyancy term to the flow shear term:

$$Ri = \frac{buoyancy; /; /term}{flow; /; /shear; /; /term}$$

Bulk Richardson number - which is an approximation of the Gradient Richardson number. The BRN is a dimensionless ratio in meteorology related to the consumption of turbulence divided by the shear production of turbulence. It is used to show dynamic stability and the formation of turbulence.

Monin-Obukhov length - which is used to describe the effects of buoyancy on turbulent flows, particularly in the lower tenth of the atmospheric boundary layer.

2.10 Mixing height and ventilation coefficients

Mixing height is measured in metres and can be stated as the height of the area above the surface of the earth wherever turbulent airflow exists. Mixing height is an important parameter when describing the air pollution climate of an area. Any pollutants or particles free into this layer will be transported and dispersed vertically (Swart, 2016). Convective heating and turbulence due to friction from the earth's surface are the key reasons for dispersion and transport in the mixing layer.

Rama et al. (2005) studied the SO_2 distribution in Venice by means of an air quality simulation model and found that the model was designed to substantially reduce the artificial diffusion which always happen when applying grid models to point sources. At every hour of simulated time, the mixing height were calculated by the model interpolating between a minimum of 200 m and a maximum of 500 m which are mostly values of Venice. It was concluded that during winter, the average concentration ranges between $20.6\mu g/m^3$ to $123.4\mu g/m^3$ and during summer, the average concentration ranges from $5.2\mu g/m^3$ to $37.9\mu g/m^3$. The results of computation indicated that the model is capable of describing the short term pollution field provided adequate detailed input parameters that are supplied.

The assimilative capacity and dispersion of pollutants due to industries in the Visakhapatnam bowl area located in the coastal Andhra Pradesh, India was studied by Rama et al. (2005). The ventilation coefficient was determined as the product of 3 hour wind speed and mixing height on each day per season and it was found to be 9781 m^2/s in summer and 13924 m^2/s in winter. The pollution potential in terms of pollutant concentrations was obtained using two dispersion models.

The different operational methods for the determination of the mixing height was compared by Ahmed & Hossain (2008). The two basic methods for the determination of the mixing height were derived profile data (measurements or numerical model output) and parameterization using simple equations or models. They concluded that all the tested parameterization schemes showed deficiencies under certain conditions, thus requiring more flexible algorithms able to take into account changing and non classical conditions.

2.11 Validation of the model

Zoras et al. (2006) studied a working model for the diffusion of gases from different sources in an urban area. The equation of diffusion which was modified to use area instead of point sources was utilized with a source inventory of SO_2 emission to compute 24 hour concentrations (0 to 12 $\mu g/m^3$ at a interval of one mile. Stability as well as wind velocity were averaged by a period of 2 hour to evaluate the equation of diffusion.

Computed concentrations more than the observed concentrations more frequently than observed concentrations for 32 stations for 35 periods.

The physical concepts underlying the measurement and modelling of pollutant concentrations caused by point sources was examined by Briggs (1975). It has been indicated that the stochastic nature of concentration fields could cause severe limitations on the ability of models to predict short term concentrations. It was shown that before comparing, it was necessary to make an estimate of the expected deviation between predictions and observations.

A Gaussian plume model was evaluated at Dickerson power plant comparing its predicted, for different methods of dispersion estimates to field measurements of SO_2 ground level concentration along the plume center line Scott & Gerhard (2002). Brookhaveh wind speed temperature gradient method, Pasquill Turner method, Pasquill Turner method dropping one stability class and TVA method were used. Each methods showed big different results for predicting dispersion. The predicted concentrations are within a factor of two of measurements in 73% of the cases studied.

CHAPTER 3

GOVERNING EQUATIONS

3.1 Introduction

All of the models are based on simple mass balance principles. A mass balance is an accounting method in which one tracks the total mass of a specie entering and leaving a system. The general balance equation is written around a specific set of boundary conditions and takes the following general form:

$$\begin{bmatrix} Accumulation \\ rate \end{bmatrix} = \begin{bmatrix} All \\ flow \\ rates \\ in \end{bmatrix} - \begin{bmatrix} All \\ flow \\ rates \\ out \end{bmatrix} + \begin{bmatrix} Creation \\ rate \end{bmatrix} - \begin{bmatrix} Distruction \\ rate \end{bmatrix} \tag{3.1.1}$$

3.2 Expressing concentration

The fundamental of attention in the field of fluid mechanics and environmental is quantity of concentration. The term concentration can be defined as a measure of the amount of a substance within a mixture (Carslaw & Jaeger, 1959). Mathematically, the concentration C is the ratio of the mass of a matter M_i to the entire volume of a mixture V stated

$$C = \frac{M_i}{V} \tag{3.2.1}$$

The units of concentration are [M/L^3], normally stated in mg/l, kg/m^3, lb/gal, ppm (parts per million), etc.

3.3 Dimensional reasoning

A very influential analytical technique that is used is dimensional analysis. The concept behind dimensional analysis is that if one can describe the parameters that a process depends on, then one should be able to custom these parameters, frequently in the method of dimensionless variables, to define that process at entirely scales (Fisher & Macqueen, 1981).

Dimensional analysis gives an influential instrument for evaluating connections amongst various parameters of a problem once the governing equation is not recognized from some additional basis, such as a theoretical consequence. The simple evidence underlying any dimensional cognitive is that all physically realistic expressions must be dimensionally dependable. In detail, the Buckingham π theorem, presented in the next section, can be understood as a proper statement of a connection amongst variables founded merely on their dimensional elements.

Some physical equation that is dimensionally composed (that is, the dimensional elements are similar for each of the terms in the equation) can be written in non-dimensional method. The calmest method to understand this is to split all the terms of the equation by one of the terms. Completed correctly, this frequently results in equations articulated in terms of mutual dimensionless constraints. Meanwhile all the terms have a similar physical dimensions, the outcome of this procedure is a relationship amongst these dimensionless variables, which can be used to assess the comparative standing of unlike terms in any assumed equation. For instance, it would be likely to gain some thoughtful of the relative significance of different forces in a specific flow field by observing at the values of the parameters in dimensionless forms of the momentum equations. This process occasionally permits simplification of a general governing equation, by removing terms that are understood as being of minor significance, matched with others.

One other significant claim of dimensional study is in providing a means of scaling the outcomes of a model study to prototype settings. This is essential, for example, in extrapolating outcomes from laboratory physical modelling studies to field conditions. In order to ensure this, conditions of similarity must be fulfilled. There are three kinds of similarity: Intuitively, a model or experiment should be geometrically alike to the field situation, which means that the ratio of all length scales is similar amongst the model and the prototype. Kinematic similarity includes likeness of length and time quantities. Dynamic likeness also must be fulfilled in order to correctly scale outcomes about forces and stresses. Kinematic and dynamic likeness is attained when suitable dimensionless parameters are identical in the model and in the prototype. Dynamic similarity is correspondent to saying the ratios of appropriate forces are similar.

3.3.1 Dimensionless parameters

Dimensional analysis is based on the theory known as Buckingham π-theorem. Let a system be defined by dimensional variables x. The given complete group of variables comprise y unique physical dimensions (temperature, time, mass, length,etc.). The Buckingham π-theorem argues that if there are $x - y$ non-dimensional that are independent, then one can come up with sets from these main variables, (Scott & Gerhard, 2002). When creating the dimensionless groups, retain the dependent variables (the one to expect) in only one of the dimensionless group.

After we have the $x - y$ dimensionless variables, the Buckingham π-theorem additional says that the variables can be associated in the following way

$$\pi_1 = f(\pi_2, \pi_i, ..., \pi_{x-y}) \tag{3.3.1}$$

such that π is considered as the i^{th} dimensionless variable.

Suppose we have to forecast the flow of fluids when it turns out to be turbulent. The variable that is dependent is a quality (turbulent or laminar) and which has no measurements (dimensions). If we consider velocity u, the flow of disturbances, it is where the variables depend which are categorized by a typical length scale L, and the fluid possessions, as defined by its dentist ρ, temperature T, and viscosity μ.

Primary, we have to know that ρ and μ are functions of T; therefore, the above three given parameters can not be preserved as independent. Hence, common dense which is an old method is by recalling the variable μ and ρ from the procedure of the kinematic viscosity $v = \frac{\mu}{\rho}$. Consequently, we have obtained $x = 3$ dimensional variables (L, u, v) in $y = 2$ physical dimensions (length and time).

From there, is to formulate the group of Dimensionless variables

$$\pi_1 = f(L, u, v)$$

This can be completed using the assumption that each parameter contains a diverse exponent where we can write each dimension with a distinct equation. What we are saying is as follows

$$\pi_1 = u^a L^b v^c \tag{3.3.2}$$

and each dimension is needed to cancel out, leaving us with two equations

$$T : 0 = -a - c$$

$$L : 0 = a + b + 2c.$$

Considering the T-equation, the following has been obtained

$$a = -c \tag{3.3.3}$$

also considering the L-equation, we obtain

$$b = -c \tag{3.3.4}$$

Subsequently, our equation are still to be defined, therefore we are allowed to select the coefficient of c.

In order to get the basic formula, we need to select

$$c = 1$$

which is giving us

$$a = b = -1$$

Therefore, it gives us

$$\pi_1 = \frac{v}{uL} \tag{3.3.5}$$

The given equation (3.3.5) of non-dimensional grouping is the inverse of the famous Reynolds number Re; therefore, we have stated through dimensional analysis, that the turbulent state of the fluid flow must rely mostly in the theory of Reynolds number, given below as

$$Re = \frac{uL}{v} \tag{3.3.6}$$

defined as a orthodox outcome in the field of fluid mechanics.

3.4 Basic transport equation

Mass balance for a control volume where the transport happens only in one direction (x-direction).

The mass balance for this situation can be written in the following procedure, (Seinfeld & Peterson, 1977):

$$\begin{bmatrix} \textit{Change of mass in} \\ \textit{the control volume} \\ \textit{in a time interval } \Delta t \end{bmatrix} = \begin{bmatrix} \textit{Mass entering} \\ \textit{the control} \\ \textit{volume in } \Delta t \end{bmatrix} - \begin{bmatrix} \textit{Mass leaving} \\ \textit{the control} \\ \textit{volume in } \Delta t \end{bmatrix} \tag{3.4.1}$$

The representation of the general transport equation in x direction is given by (Crank, 1975)

$$\frac{\partial C}{\partial t} = \frac{1}{\Delta x}.(J_1 - J_1 - \frac{\partial J}{\partial x}\Delta x) \tag{3.4.2}$$

$$\frac{\partial C}{\partial t} = -\frac{\partial J}{\partial x} \tag{3.4.3}$$

where C is the concentration over time t and J is the flux. The flux is changing in x direction with the gradient of $\frac{\partial J}{\partial x}$.

We are existing in a three-dimensional space, where the similar rules for the general mass balance and transport are binding in all dimensions. Consequently,

$$\frac{\partial C}{\partial t} = -\sum_{i=1}^{3} \frac{\partial}{\partial x_i} J_i \tag{3.4.4}$$

where

$$x_1 = x$$
$$x_2 = y$$
$$x_3 = z$$

which yields to

$$\frac{\partial C}{\partial t} = -(\frac{\partial}{\partial x} J_x + \frac{\partial}{\partial y} J_y + \frac{\partial}{\partial z} J_z) \tag{3.4.5}$$

The transport equation is derived for a conservative tracer or substantial. The control volume is constant as the time advancements. The flux (J) can be everything (i.e. flows, dispersion, etc).

3.5 Advective flux

The advective flux can be analysed with the simple theoretical model, which comprises two control volumes. Advection happens only towards one direction in a time intermission. The equation for Advective flux is given by, (Settles, 2006)

$$J_A = \frac{\partial x}{\partial t} C \tag{3.5.1}$$

where C is concentration and J is the advective flux.

3.6 Diffusion

Diffusion is the most important transport process in the field of environmental fluid mechanics. Diffusion varies from advection in a manner it is random in nature (does not necessarily follow a fluid particle). Transfer of heat by conduction is due to random molecular motions, and there is an understandable similarity among the two processes. This was documented by Fick (1855), who initially put diffusion on a quantitative basis by adopting the mathematical equation of heat conduction resulting some years earlier by Fourier (1822). The mathematical theory of diffusion in isotropic substance is hence founded on the hypothesis that the rate of transfer of diffusing substance through the unit area of a section is proportional to the concentration gradient measured normal to the section, (Crank, 1975):

$$J_D = -D \frac{\partial C}{\partial x} \tag{3.6.1}$$

where C is the concentration of diffusing substance, x the space coordinate measured normal to the section and D is called the diffusion coefficient. The negative sign arises because diffusion occurs in the direction opposite to that of increasing concentration. It must be emphasized that the statement expressed mathematically by equation (3.6.1) is in general consistent only for an isotropic medium, whose structure and diffusion properties in the neighbourhood of any point are the same ralative to all directions. Equation (3.6.1) is called Fick's first law of diffusion, x direction.

Assuming conservation of mass, then

$$\frac{\partial c}{\partial t} + \frac{\partial J}{\partial x} = 0 \tag{3.6.2}$$

$$\frac{\partial c}{\partial t} - \frac{\partial}{\partial x}(D \frac{\partial c}{\partial x}) = 0 \tag{3.6.3}$$

Rearranging the above equation and simplify, gives

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \tag{3.6.4}$$

This is Ficks second law of Diffusion (one dimensional).

In three dimensions we have (Ahmad, 2000):

$$\frac{\partial c}{\partial t} = D[\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2}] \tag{3.6.5}$$

3.7 Advection Diffusion Equation.

In the environment, the movement of fluids happens by combining advection as well as diffusion together. To derive advection diffusion equation, we need to employ Fick's law of diffusion and the conservation of mass. Recall the following equations from above:

$$\frac{\partial C}{\partial t} = -\frac{\partial J}{\partial x} \tag{3.7.1}$$

$$J_A = \frac{\partial x}{\partial t} C \tag{3.7.2}$$

$$J_D = -D\frac{\partial C}{\partial x} \tag{3.7.3}$$

Therefore, the total flux is given as,

$$J = J_A + J_D \tag{3.7.4}$$

Due to conservation of mass,

$$\frac{\partial c}{\partial t} + \frac{\partial J}{\partial x} = 0 \tag{3.7.5}$$

Substitution in equation (3.7.5) and simplify, yields

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + D\frac{\partial^2 C}{\partial x^2} \tag{3.7.6}$$

Equation (3.7.6) can be written in three dimensions as

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = K\frac{\partial^2 C}{\partial x^2} + K\frac{\partial^2 C}{\partial y^2} + K\frac{\partial^2 C}{\partial z^2} \tag{3.7.7}$$

with D replaced by K. This is the equation for heat conduction in a solid with the variables renamed. Our immediate reaction is to factor K out of the three terms on the right, but the data from experimental method states that for turbulent diffusion in the air, the values of K in the three dimensions are not the same. So, in subsequent equations, we will write the three Ks as

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = K_x\frac{\partial^2 C}{\partial x^2} + K_y\frac{\partial^2 C}{\partial y^2} + K_z\frac{\partial^2 C}{\partial z^2} \tag{3.7.8}$$

Equation (3.7.8) is called Advection Diffusion Equation in three dimensions. Remember that the above equations use the assumption that the value of K is constant throughout.

Next we make assumptions to simplify the Advection Diffusion equation.

- Unsteady state situation is observed (i.e. $\frac{\partial C}{\partial t} \neq 0$).
- The vertical velocity component (v) and horizontal velocity component (w) is ignored.
- the x-axis is aligned in the direction of mean wind (i.e. $u = U$, $v = 0$).
- movement due to mean wind is ignored compared to diffusion as result of downwind (i.e. $\mid K_x \frac{\partial^2 C}{\partial x^2} \mid \gg \mid U \frac{\partial C}{\partial x} \mid$).

Application of these assumptions in equation (3.7.8), leads to unsteady state advection diffusion equation for dispersion of a contaminant released from point source in three dimensions as (Seinfeld, 1986):

$$\frac{\partial C}{\partial t} = K_x \frac{\partial^2 C}{\partial x^2} + K_y \frac{\partial^2 C}{\partial y^2} + K_z \frac{\partial^2 C}{\partial z^2} \tag{3.7.9}$$

given that x, y and z are coordinates of wind in the x direction, y direction and z direction respectively. C is given as the concentration of pollutants. The eddy diffusivities of pollutants in the x direction, y direction and z directions are given as K_x, K_y and K_z respectively.

3.8 Gaussian plume derivation

There are numerous approaches that can be used to explain equation (3.8.1). In this thesis, we used two methods for the derivations of Gaussian Plume Equation. The first method is Similarity Solution and the second is Laplace Transform. Similarity solutions to PDEs are solutions which depend on certain groupings of the independent variables, instead on each variable separately (Lin & Hildemann, 1996).

3.8.1 Solution using Similarity method to 1-D diffusion equation.

Suppose we have a one dimensional diffusion equation for (3.7.9) as

$$\frac{\partial C}{\partial t} = K_x \frac{\partial^2 C}{\partial x^2} \tag{3.8.1}$$

We selected a method known as similarity technique for us to demonstrate the importance of dimensional analysis outlined in 3.3.

Let us have a look at a one dimensional scenario of a thin, unlimited cylinder (radius a) as shown in Figure 3.1 underneath.

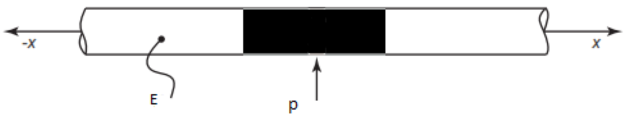


Figure 3.1: One dimensional pure diffusion in an unlimited cylinder, (Bluett et al., 2004)

So, a mass of pollutant P is released homogeneously over the cross section of area

$$E = \pi r^2$$

at a given point

$$x = 0$$

with time

$$t = 0$$

The primary width of the pollutant is infinitesimally trivial. We are looking for the solution for the spread of pollutant in time due to molecular diffusion only.

The above given problem is a 1-D unsteady diffusion,i.e.

$$\frac{\partial C}{\partial y} = 0$$

also

$$\frac{\partial C}{\partial z} = 0$$

So equation (3.8.1) is the main equation, and we need two boundary conditions and an initial condition. For boundary conditions, we execute that

$$C(\infty, t) = 0 \tag{3.8.2}$$

and

$$C(-\infty, t) = 0 \tag{3.8.3}$$

The initial condition is given that the pollutant is released homogeneously over the cross section across an infinitesimally small width in the x- direction is the initial condition. For such an initial condition to be useful, Dirac Delta function is employed

$$C(x, 0) = \frac{P}{E}\delta(x) \tag{3.8.4}$$

where

$$\delta(x) = 0$$

every where excluding where

$$x = 0$$

since it is infinite, then again,

$$\int_{-\infty}^{\infty} \delta(x)dx = 1$$

Consequently, the total released mass is assumed to be

$$P = \int_{v} C(x,t)dv \tag{3.8.5}$$

$$P = \int_{-\infty}^{\infty} \int_{0}^{r} \frac{P}{E}\delta(x)2\pi r dr dx \tag{3.8.6}$$

To use dimensional analysis technique, its necessity we contemplate all the parameters that control the solution. Table below summarises the dependent and independent variables for our problem.

Table 3.1: Dimensional variables for 1-D of diffusion in a cylinder.

	Variable	Dimensions
dependent variable	C	P/L^3
independent variables	P/E	P/L^2
	D	L^2/T
	x	L
	t	T

Here are $y = 3$ dimensions and $x = 5$ parameters; therefore, one has to formulate two groups of dimensionless

$$\pi_1 = \frac{C}{P/(E\sqrt{Kt})} \tag{3.8.7}$$

$$\pi_2 = \frac{x}{\sqrt{Kt}} \tag{3.8.8}$$

Based on dimensional analysis technique, we obtained

$$\pi_1 = f(\pi_2)$$

hence it suggests that for the solution of C

$$C = \frac{P}{E\sqrt{Kt}} f\left(\frac{x}{\sqrt{Kt}}\right) \tag{3.8.9}$$

such that f is a function which is not yet known with π_2 as the argument. Equation (3.8.9) is called a similarity solution because C has the similar shape in x at all times t. Next we are required to discovery f for us to identify its shape.

There are two primary methods in which we can found the function f. We can solve f analytically and use equation (**3.8.9**) as the solution to a differential equation. The supremacy of similarity solution is that it changes a partial differential equation (PDE) into an ordinary differential equation (ODE), which is the objective of every solution methods for PDEs.

The similarity solution (3.8.9) is actually just the transformation of coordinate. Therefore, we will demand our new similarity variable to be defined as

$$\eta = \frac{x}{\sqrt{Kt}}$$

In order to put equation (3.8.9) in the equation of diffusion, the following two derivatives is required

$$\frac{\partial \eta}{\partial t} = -\frac{\eta}{2t} \tag{3.8.10}$$

$$\frac{\partial \eta}{\partial x} = \frac{1}{\sqrt{Kt}} \tag{3.8.11}$$

Firstly, we employ chain rule to calculate $\frac{\partial C}{\partial t}$ as given below

$$\begin{aligned}\frac{\partial C}{\partial t} &= \frac{\partial}{\partial t}\left[\frac{P}{E\sqrt{Kt}} f(\eta)\right] \\ &= \frac{\partial}{\partial t}\left[\frac{P}{E\sqrt{Kt}}\right] f(\eta) + \frac{P}{E\sqrt{Kt}} \frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial t} \\ &= \frac{P}{E\sqrt{Kt}}\left(-\frac{1}{2}\right)\frac{1}{t} f(\eta) + \frac{P}{E\sqrt{Kt}}\frac{\partial f}{\partial \eta}\left(-\frac{\eta}{2t}\right) \\ &= -\frac{P}{2Et\sqrt{Kt}}\left(f + \eta\frac{\partial f}{\partial \eta}\right) \end{aligned} \tag{3.8.12}$$

Likewise, we apply chain rule to calculate $\frac{\partial^2 C}{\partial x^2}$ which yields to

$$\begin{aligned}\frac{\partial^2 C}{\partial x^2} &= \frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left(\frac{P}{E\sqrt{Kt}} f(\eta)\right)\right] \\ &= \frac{\partial}{\partial x}\left[\frac{P}{E\sqrt{Kt}}\frac{\partial \eta}{\partial x}\frac{\partial f}{\partial \eta}\right] \\ &= \frac{P}{EKt\sqrt{Kt}}\frac{\partial^2 f}{\partial \eta^2} \end{aligned} \tag{3.8.13}$$

Upon replacing these two outcomes into the diffusion equation, we get the ordinary differential equation in η

$$\frac{d^2 f}{d\eta^2} + \frac{1}{2}\left(f + \eta\frac{df}{d\eta}\right) = 0 \tag{3.8.14}$$

To solve equation (3.8.14), we must also change the boundary and initial conditions to two new constraints on f. The initial conditions and boundary conditions both can be content through a single condition on f. Considering conservation of mass, the other constraint is taken from and is given by equation (3.8.5).

Substituting

$$dx = d\eta\sqrt{Kt}$$

into equation (3.8.5) and simplifying, we get

$$\int_{-\infty}^{\infty} f(\eta)d\eta = 1 \tag{3.8.15}$$

Solving equation (3.8.14) involves couple of integrations. Foremost, we reorganize the equation by means of the identity

$$\frac{d(f\eta)}{d\eta} = f + \eta\frac{df}{d\eta} \tag{3.8.16}$$

which gives us

$$\frac{d}{d\eta}\left[\frac{df}{d\eta} + \frac{1}{2}f\eta\right] = 0 \tag{3.8.17}$$

If we integrate once, we obtain

$$\frac{df}{d\eta} + \frac{1}{2}f\eta = C_0 \tag{3.8.18}$$

Both boundary conditions and the initial condition can be satisfied by taking $C_0 = 0$.

If we consider $C_0 = 0$, we get a homogeneous ordinary differential equation whose solution can readily be established. If we move the second term to the right hand side of the equation, we obtain

$$\frac{df}{d\eta} = -\frac{1}{2}f\eta \tag{3.8.19}$$

The result is found by accumulating the f and η terms on separate sides of the equation

$$\frac{df}{f} = -\frac{1}{2}\eta d\eta \tag{3.8.20}$$

Integrating both sides yields

$$ln(f) = -\frac{1}{2}\frac{\eta^2}{2} + C_1 \tag{3.8.21}$$

Subsequently, taking the exponential of both sides yields

$$f = C_1 exp\left(\frac{\eta^2}{4}\right) \tag{3.8.22}$$

To discover C_1 it is essential to use the remaining constraint specified in equation (3.8.15)

$$\int_{-\infty}^{\infty} C_1 exp\left(-\frac{\eta^2}{4}\right) d\eta = 1 \tag{3.8.23}$$

In order to compute the above integral, one must employ the integral tables; consequently, we need to make one additional change of variables in order to eliminate the $\frac{1}{4}$ from the exponential. Therefore, we institute ζ defined as

$$\zeta^2 = \frac{1}{4}\eta^2 \tag{3.8.24}$$

$$2d\zeta = d\eta \tag{3.8.25}$$

Replacing the above transformation of coordinate and compute C_1 yields

$$C_1 = \frac{1}{2\int_{-\infty}^{\infty} exp(-\zeta^2)d\zeta} \tag{3.8.26}$$

Subsequently, if we look up in the integral table, yields to

$$C_1 = \frac{1}{2\sqrt{\pi}}$$

Hence,

$$f(\eta) = \frac{1}{2\sqrt{\pi}} exp\left(\frac{\eta^2}{4}\right) \tag{3.8.27}$$

Substituting f in our similarity solution (3.8.9) yields

$$C(x,t) = \frac{P}{E\sqrt{4\pi Kt}} exp\left(-\frac{x^2}{4Kt}\right) \tag{3.8.28}$$

an equation which was utilised throughout this thesis and is also a classic result in environmental fluid mechanics. In three dimensions, we can generalise to give the solution Scott & Gerhard (2002)

$$C(x,y,z,t) = \frac{P}{E\sqrt{4\pi K_x K_y K_z}} exp\left(-\frac{x^2}{4K_x t} - \frac{y^2}{4K_y t} - \frac{z^2}{4K_z t}\right) \tag{3.8.29}$$

and has been derived through the method of separation of variables. Equation (3.8.28) is still in the Lagrangian coordinates x, y and z and are measured from the center of mass of the cloud. Center of mass assumed to move with wind speed and direction (distance travelled = ut).

Equation (3.8.29) is normally known as the *Gaussian plume solution* for the advection-diffusion equation, given that we can compare the exponential dependence on both y and z to that of a Gaussian-type function.

3.8.2 Solution using Laplace Transform

The following derivations were borrowed from The Mathematics of Atmospheric Dispersion Modelling Journal (Briggs, 1975) . Our attention is more on the movement of a one pollutant such that the mass concentration at a given location

$$(x, y, z) \in \mathbb{R}^3$$

given in time

$$t \geqslant 0 \quad [s]$$

which is given by an even function

$$C(x, y, z, t)$$

We demonstrated the law of conservation of mass for C in differential form, given as

$$\frac{\partial C}{\partial t} + \nabla.\vec{J} = S \tag{3.8.30}$$

given that

$$S(x, y, z, t)$$

is defined as source term and the vector function is given as

$$\vec{J}(x, y, z, t)$$

where $[kg/m^2s]$ acts for the mass flux of contaminant unsettled to the combined consequences of diffusion and meteorology. The scattering donation to the flux arises from the unstable eddy motion in the atmospheric surrounding. The primary outcome is that the atmospheric diffusion may presume to follow Ficks law, which states, "the diffusive flux is proportional to the concentration gradient" or

$$\vec{J}_D = -K\nabla C$$

The negative results guarantee that the contaminant flows from regions of high concentration to regions of low concentration, and the diffusion coefficient

$$(K_x, K_y, K_z) \quad [m^2/s]$$

is a turbulent eddy diffusivity of which in general is the purpose of position. The second offering to the flux is because of simple linear convection by the wind, which can be expressed as

$$\vec{J}_A = C\vec{u}$$

whereas

$$\vec{u} = (u, v, w) \;\; [m/s]$$

is the wind velocity. By adding these various benefactions together, we obtain the total flux

$$\vec{J} = \vec{J}_D + \vec{J}_A = C\vec{u} - K\nabla C$$

which after replacements into the equation of conservation of mass (3.6.1) yields the three-dimensional convection-dispersal equation

$$\frac{\partial C}{\partial t} + \nabla.(C\vec{u}) = \nabla.(K\nabla C) + S \tag{3.8.31}$$

Furthermore, a number of clear supposition that will allow us to obtain a closed-form analytical solution:

1. The contaminant is emitted at a constant rate Q [kg/s] from a single point source

   $$(x, y, z) = (0, 0, H)$$

   after placed at a high point H above the ground surface. Then the native term can be given as

   $$S(x, y, z) = Q\delta(x)\delta(y)\delta(z - H) \tag{3.8.32}$$

   given that $\delta(.)$ is the Dirac delta function. It should be eminent here that m^{-1} is the units of the delta function. For the stack-like configuration presented in Figure 2.2 the height is actually a functional height, given as $H = h + \delta h$, which is the total sum of the true stack height h and the plume rise δh that comes from buoyant effects.

2. Wind velocity is taken to be constant and aligned with the positive x-axis and its defined as

   $$(u, v, w) = (u, 0, 0)$$

   such that $u \geqslant 0$ for the given some constant. We rest this assumption is rested for later to permit a universal time-varying wind field

   $$(u, v, w) = (u, v, w)(t),$$

   for

   $$\mid (u, v, w) \mid \geqslant 0.$$

3. It is stated here that a steady state solution is considered, so that if the wind velocity and various other variables are taken to be independent of time and the time range of interest is sufficiently long which is reasonable enough.

4. The diffusivities of eddy motion are functions of the wind in the x from the source only, while diffusion is taken to be isotropic so that

   $$K_x(x) = K_y(x) = K_z(x) = K(x)$$

5. Wind velocity is large enough that advection is much larger than diffusion in the x-direction is much smaller and hence the term

   $$K_x\partial_x^2 C$$

   can be overlooked.

6. If we ignore the variations in topography, then the surface of the ground can be taken as $z = 0$ in a plane.

7. The pollutants should not go through the ground.

If we make use of the above simplifying assumptions from 1 to 6, equation (3.6.2) yields to

$$u\frac{\partial C}{\partial x} = K\frac{\partial^2 C}{\partial y^2} + K\frac{\partial^2 C}{\partial z^2} + Q\delta(x)\delta(y)\delta(z-H) \tag{3.8.33}$$

and we are much more interested in the result for values of $x, z \in [0, \infty)$ as well as $y \in (-\infty, \infty)$. For us to get a well-modelled problem, a Partial Differential Equation (PDE) must be supplemented with an appropriate group of boundary conditions, that is,

$$C(0, y, z) = 0 \tag{3.8.34}$$

$$C(\infty, y, z) = 0 \tag{3.8.35}$$

$$C(x, \pm\infty, z) = 0 \tag{3.8.36}$$

$$C(x, y, \infty) = 0 \tag{3.8.37}$$

Condition number one is a result of the unidirectional wind and we have assumed that there are no contaminant sources if the values of $x < 0$. The last three boundary conditions at infinity agreeing with the condition that the total mass of contaminant should remain finite.

Assumption 7 states that, the vertical flux at the ground should disappear, giving us the last boundary condition

$$K\frac{\partial C}{\partial z}(x, y, 0) = 0 \tag{3.8.38}$$

When taken together, equation (3.8.33) to (3.8.37) is a well-defined problem for the steady-state contaminant concentration.

$$C(x, y, z)$$

The solution to this problem can be obtained by removing the source term from the Partial Differential Equation and rather bringing a delta function term into the boundary condition:

$$u\frac{\partial C}{\partial x} = K\frac{\partial^2 C}{\partial y^2} + K\frac{\partial^2 C}{\partial z^2} \tag{3.8.39}$$

$$C(0, y, x) = \frac{Q}{u}\delta(y)\delta(z-H) \tag{3.8.40}$$

$$C(\infty, y, z) = 0, \quad C(x, \pm\infty, z) = 0, \quad C(x, y, \infty) = 0 \tag{3.8.41}$$

$$K\frac{\partial C}{\partial z}(x, y, 0) = 0 \tag{3.8.42}$$

The eddy diffusion coefficients in the atmospheric boundary layer are strong functions of downwind distance, not to mention that they vary with weather conditions and time from release, and consequently they are difficult to find in practice. Therefore, in most literature, the variable x has been replaced with the new independent variable given as

$$r = \frac{1}{u}\int_0^x K(\xi)d(\xi) \tag{3.8.43}$$

and has units of $[m^2]$. Later we will observe that the variable r is a fixed multiple of the variance of the concentration distribution, which is proposed and be denoted by σ. In equation (3.8.39), the K coefficients is eliminated through change of variables, which leads to the following constant coefficient problem for

$$\frac{\partial c}{\partial r} = \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} \tag{3.8.44}$$

From equation (3.8.40 - 3.8.42), the boundary conditions for c are identical to those for C except that x is replaced with r. Next we use the method of separation of variables to equation (3.8.44), with the assumption that the dependence of the solution on y and z can be separated according to (This is a minor change of the regular division of variables approach where we routinely presume an entirely separation solution of the form) $c(r, y, z) = R(r)Y(y)Z(z)$):

$$c(r, y, z) = \frac{Q}{u} a(r, y).b(r, z) \tag{3.8.45}$$

Two reduced dimension problems was obtained which has a form of two-dimensional diffusion equations given as:

$$\frac{\partial a}{\partial r} = \frac{\partial^2 a}{\partial y^2} \tag{3.8.46}$$

for

$$0 \leq r < \infty$$

and

$$-\infty < y < \infty$$

with the boundary conditions

$$a(0, y) = \delta(y), \quad a(\infty, y) = 0, \quad a(r, \pm\infty) = 0 \tag{3.8.47}$$

as well as

$$\frac{\partial b}{\partial r} = \frac{\partial^2 b}{\partial z^2} \tag{3.8.48}$$

for

$$0 \leq r < \infty$$

and

$$0 < z < \infty$$

with the boundary conditions

$$b(0, z) = \delta(z - H), \quad b(\infty, z) = 0, \quad b(r, \infty) = 0, \quad \frac{\partial b}{\partial z}(r, 0) = 0 \tag{3.8.49}$$

The variable r in all problems above can be seen as a time-like variable, so the boundary conditions at $r = 0$ (which contain the delta functions) behave as initial conditions for the given diffusion equations.

The Laplace transform can be used to solve differential equations. Besides being a different and efficient alternative to variation of parameters and undetermined coefficients, the methods of Laplace is significant for input terms that are piecewise defined, periodic or impulsive.

Assume $f(t)$ is defined for $t \geqslant 0$. The Laplace transform of $f(t)$, represented by $F(s)$ or $\mathcal{L}\{f(t)\}$, is an integral transform specified by the Laplace integral:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}dt \tag{3.8.50}$$

on condition that this integral exists, i.e. that the integral is convergent.

The theorems we need are given below; in most cases we indicate the proofs briefly without stating the conditions.

Theorem 1.

$$\mathcal{L}\left\{\frac{\partial v}{\partial t}\right\} = \rho\mathcal{L}\{v\} - v_0 \tag{3.8.51}$$

where v_0 is the value of

$$\lim_{t\to 0} v.$$

In general v_0 will be a function of the space variables x, y, z. The proof for equation (3.8.51) follows immediately on integrating by parts, since

$$\int_0^\infty e^{-\rho t}\frac{\partial v}{\partial t}dt = \left[e^{-\rho t}v\right]_0^\infty + \rho\int_0^\infty e^{-\rho t}dt = -v_0 + \rho\bar{v}.$$

Theorem 2

$$\mathcal{L}\left\{\frac{\partial^n v}{\partial x^n}\right\} = \frac{\partial^n \bar{v}}{\partial x^n} \tag{3.8.52}$$

with similar results from the other space variables. This is equivalent to

$$\int_0^\infty e^{-\rho t}\frac{\partial^n v}{\partial x^n}dt = \frac{\partial^n}{\partial x^n}\int_0^\infty e^{-\rho t}vdt$$

and we assume v to be such that the orders of integration and differentiation can be interchanged in this way.

Theorem 3

suppose $f(t)$ is continuous and has a piece-wise continuous derivative $f^{'}(t)$. Then by integration by parts,

$$\mathcal{L}\{f^{'}(t)\} = \int_0^\infty f^{'}(t)e^{-st}dt$$

$$\mathcal{L}\{f^{'}(t)\} = e^{-st}f(t)\mid_0^\infty + s\int_0^\infty f(t)e^{-st}dt \tag{3.8.53}$$

If $f(t)$ is of exponential order[2], then $e^{-st}f(t)$ tends to zero as $t \to \infty$, for large enough s, so that

$$\mathcal{L}\{f^{'}(t)\} = sF(s) - f(0) \tag{3.8.54}$$

Similarly, if $f(t)$ and $f^{'}(t)$ are continuous, $f^{''}(t)$ is piece-wise continuous, and all three functions are of exponential order, then

$$\mathcal{L}\{f^{''}(t)\} = s\mathcal{L}\{f^{'}(t)\} - f^{'}(0) = s^2F(s) - f(0) - f^{'}(0) \tag{3.8.55}$$

$Theorem$ 4

If $f(t) = H(t-t_0)\phi(t-t_0)$, where $H(t-t_0)$ is $Heaviside's\ unit\ function\ defined\ by$

$$H(t-t_0) = 0, t < t_0$$
$$H(t-t_0) = 1, t > t_0$$

then

$$\mathcal{L}\{f(t)\} = e^{-\rho t_0}\mathcal{L}\{\phi(t)\} \tag{3.8.56}$$

In equation (3.8.46), we start with the problem for $a(r,y)$ and use the Laplace transform of the Partial Differential Equation in r to obtain

$$\rho\hat{a} - a(0,y) = \frac{\partial^2\hat{a}}{\partial y^2} \tag{3.8.57}$$

whereas

$$\hat{a}(\rho,y) := \mathcal{L}_r\{a(r,y)\} = \int_0^\infty e^{-\rho r}a(r,y)dr \tag{3.8.58}$$

and here ρ is the modifiable unknown term.

To get the next ordinary differential equation (ODE) for $\hat{a}$, we need to use the source boundary condition (3.8.47):

$$\frac{\partial^2\hat{a}}{\partial y^2} - \rho\hat{a} = -\delta(y) \tag{3.8.59}$$

If we use the Laplace transform in y, yields

$$\eta^2\hat{\hat{a}} - \eta\hat{a}(\rho,0) - \frac{\partial\hat{a}}{\partial y}(\rho,0) - \rho\hat{\hat{a}} = -1 \tag{3.8.60}$$

given that

$$\hat{a}(\rho,\eta) := \mathcal{L}_y\{\hat{a}(\rho,y)\} = \int_0^\infty e^{-\eta y}\hat{a}(\rho,y)dy \tag{3.8.61}$$

and η is known as a transform variable.

Even though the symmetry permits the solution to be extended over the whole range of $-\infty < y < \infty$, at this time, we limit our problem to values of $0 \leqslant y < \infty$.

Solving equation (3.8.61), yields

$$\hat{a}(\rho,\eta) = \frac{\eta c_1 + c_2}{\eta^2 - \rho} \tag{3.8.62}$$

under condition that

$$\begin{aligned} c_1 &= \hat{a}(\rho,0) \\ c_2 &= \partial_y \hat{a}(\rho,0) - 1 \end{aligned}$$

Next we use the inverse transform in η which results in

$$\begin{aligned} \hat{a}(\rho,y) &= c_1\cosh(\sqrt{\rho}y) - \frac{c_2}{\sqrt{\rho}}\sinh(\sqrt{\rho}y) \\ \hat{a}(\rho,y) &= \frac{c_1}{2}\left(e^{\sqrt{\rho}y} + e^{-\sqrt{\rho}y}\right) - \frac{c_2}{2\sqrt{\rho}}\left(e^{\sqrt{\rho}y} - e^{-\sqrt{\rho}y}\right) \end{aligned} \tag{3.8.63}$$

in order that

$$\hat{a} \longrightarrow 0$$

as

$$y \longrightarrow \infty$$

it is necessary that

$$c_1 = \frac{c_2}{\sqrt{\rho}}$$

after which the given formula for $\hat{a}$ minimize to

$$\hat{a}(\rho,y) = \frac{c_2}{\sqrt{\rho}}e^{-\sqrt{\rho}y} \tag{3.8.64}$$

If we assume for the mean time that c_2 is self-reliant and not contingent on ρ, then we may use the inverse transform in ρ to obtain

$$a(r,y) = \left(\frac{c_2}{\sqrt{\pi r}}\right)e^{\left(-\frac{y^2}{4r}\right)} \tag{3.8.65}$$

Using the delta function identity

$$\delta(y) = \lim_{r \longrightarrow 0} e^{\left(-\frac{y^2}{4r}\right)} \frac{1}{\sqrt{4\pi r}} \tag{3.8.66}$$

We concluded that $c_2 = \frac{1}{2}$ is true a constant and that

$$a(r, y) = \frac{1}{\sqrt{4\pi r}} e^{\left(-\frac{y^2}{4r}\right)} \tag{3.8.67}$$

In order to use Laplace transforms, we have to limit ourselves to $0 \leqslant y < \infty$. As stated in equation (3.8.46 - 3.8.47), clearly has even symmetry about $y = 0$, and what we obtain in equation (3.6.27) is also an even function, then it is possible to simply extend the domain of validity for $a(r, y)$ to $y \in (-\infty, \infty)$.

Using the Laplace transform in r of the Partial Differential Equation to the solution of equations (3.8.48 - 3.8.49) for $b(\rho, z)$, yields to

$$\frac{\partial^2 \hat{b}}{\partial z^2} - \rho \hat{b} = -\delta(z - H) \tag{3.8.68}$$

where

$$\hat{b}(\rho, z) := \mathcal{L}_r\{b(r, z)\} = \int_0^\infty e^{-\rho r} \hat{b}(r, z) dr \tag{3.8.69}$$

If we consider taking the Laplace transform in z and defining

$$\hat{\hat{b}}(\rho, \zeta) := \mathcal{L}_z\{\hat{b}(\rho, z)\}, \tag{3.8.70}$$

we realise that

$$\zeta^2 \hat{\hat{b}} - \zeta \hat{b}(\rho, 0) - \frac{\partial \hat{b}}{\partial z}(\rho, 0) - \rho \hat{\hat{b}} = -e^{-\zeta H} \tag{3.8.71}$$

Subsequently, applying the changed Neumann boundary condition

$$\partial_z \hat{b}(\rho, 0) = 0$$

we can work out for

$$\hat{\hat{b}}(\rho, \zeta) = \frac{\zeta \hat{b}(\rho, 0) - e^{-\zeta H}}{\zeta^2 - \rho} \tag{3.8.72}$$

and impose the inverse transform in ζ to get

$$\hat{b}(\rho, z) = \hat{b}(\rho, 0) \cosh(\sqrt{\rho} z) - \frac{1}{\sqrt{\rho}} \sinh(\sqrt{\rho}(z - H)) \tag{3.8.73}$$

We then foist the condition that

$$\hat{b} \longrightarrow 0$$

as

$$z \longrightarrow \infty$$

which means that

$$\hat{b}(\rho, 0) = \frac{e^{-\sqrt{\rho}H}}{\sqrt{\rho}} \tag{3.8.74}$$

and therefore

$$\hat{b}(\rho, z) = \frac{1}{2\sqrt{\rho}}(e^{-\sqrt{\rho}(z-H)} + e^{-\sqrt{\rho}(z+H)}) \tag{3.8.75}$$

Lastly, applying the inverse transform in ρ, we obtain

$$b(r, z) = \frac{1}{\sqrt{4\pi r}} \left(e^{\left(-\frac{(z-H)^2}{4r}\right)} + e^{\left(-\frac{(z+H)^2}{4r}\right)} \right) \tag{3.8.76}$$

The contaminant concentration can now be determined by substituting equation (3.8.67) and (3.8.76) into equation (3.8.45), which yields to:

$$c(r, y, z) = \frac{Q}{4\pi u r} e^{\left(\frac{y^2}{4r}\right)} \left[e^{\left(-\frac{(z-H)^2}{4r}\right)} + e^{\left(-\frac{(z+H)^2}{4r}\right)} \right] \tag{3.8.77}$$

Equation (3.8.77) is usually known as the Gaussian plume solution for the advection-diffusion equation, under condition that the exponential dependence on both y and z is the same to that of a Gaussian-type function.

3.9 Gaussian Plume, 2 Dimensional spreading.

From equation (3.8.77), the 2 Dimensional spread is given as

$$C(y, z, t) = \frac{P}{4\pi t\sqrt{K_y K_z}} exp\left(-\frac{y^2}{4K_y t} - \frac{z^2}{4K_z t}\right) \tag{3.9.1}$$

Since mass is moving with the wind, mass transfer between adjacent sheets is essentially zero. Convert a z dimensional problem i.e. spread (dispersion) is going in the y and z directions.

Time for sheet to move across smoke stack/ pipe i.e.

$$\frac{m}{um/s} = t = \frac{1}{u} \textbf{ seconds}$$

so that the amount of pollutant originally injected into the sheet we are considering is

$$P = \frac{Q}{u} \tag{3.9.2}$$

where u = wind velocity. $\frac{Q}{u}$ has dimensions of mass/length, i.e. the amount injected per unit length of air passing over the stack.

Some material balances and replacements has been done so that we can use the 2 dimensional equation (3.9.1) for solving problems. Substitution back yields,

$$C(y,z,t) = \frac{Q/u}{4\pi\frac{1}{u}\sqrt{K_y K_z}} exp\left(-\frac{y^2}{4K_y\frac{1}{u}} - \frac{z^2}{4K_z\frac{1}{u}}\right) \tag{3.9.3}$$

Historically, substitute the following into equation 3.9.3 2-D turbulent diffusion equation:

$$K_y = 0.5\sigma_y^2\left(\frac{u}{x}\right) \tag{3.9.4}$$

$$K_z = 0.5\sigma_z^2\left(\frac{u}{x}\right) \tag{3.9.5}$$

$$t = \frac{x}{u} \tag{3.9.6}$$

Substituting the above equations (3.9.4 - 3.9.6) into equation 3.9.2, we have

$$C(y,z) = \frac{P}{2\pi u\sigma_y\sigma_z} e^{\left(-\frac{y^2}{2{\sigma^2}_y} - \frac{z^2}{2{\sigma^2}_z}\right)} \tag{3.9.7}$$

where σ_y is a horizontal dispersion coefficients while σ_z is a vertical dispersion coefficients. The unit for the above coefficients are given in meters (m).

If we set

$$z = (z - H) \tag{3.9.8}$$

and

$$P = Q \tag{3.9.9}$$

then equation (3.9.7) becomes

$$C(y,z) = \frac{Q}{2\pi u\sigma_y\sigma_z} e^{\left(-\frac{y^2}{2{\sigma^2}_y} - \frac{(z-H)^2}{2{\sigma^2}_z}\right)} \tag{3.9.10}$$

Equation (3.9.10) is the basic Gaussian Plume Equation. The basic Gaussian plume equation predicts a plume that is symmetrical with respect to y and with respect to z. Different values of σ_y and σ_z mean that spreading in the vertical and horizontal directions are not equal. Most often σ_y is greater than σ_z so that a contourof constant concentration is like an ellipse, with the long axis horizontal. Close to the ground this symmetry is disturbed.

To use the Gaussian plume equation one must know the appropriate values of σ_y and σ_z. From equations (3.9.4) and (3.9.5) we would expect them to have the form

$$\sigma_y = (\frac{2K_y x}{u})^{1/2} \tag{3.9.11}$$

Concentration on the centreline of the plume is given by the initial term. The two dispersion coefficient values upsurge with downwind distance, so that this centreline concentration decreases with the downwind distance. Concentration decreases as we move horizontally and vertically away from the centreline which is demonstrated by the second and third terms. Substitutions convert equation (3.9.10) to Eulerian Viewpoint. Coordinates are now measured from smoke stack (not center of mass of plume).

3.10 Some Modifications of the Basic Gaussian Plume Equation

3.10.1 The effect of the ground

At present, equation (3.9.10) is our best simple prediction method for the concentration in plumes considerable distances above the ground. However, we are generally most interested in concentrations at ground level because that is where most people and property are exposed. The blind application of equation (3.9.10) at or near ground level gives misleadingly low results. It indicates that pollutants continue to disperse at any value of z, even at z less than zero.

The ground damps out vertical dispersion. The upward and downward random atmospheric eddies that spread the plume in the vertical direction cannot penetrate the ground. Thus, vertical spreading terminates at ground level. To account for this in calculations it is commonly assumed that any pollutants that would have carried below $z = 0$ if the ground were not there are reflected upward as if the ground were a mirror. Thus, the concentration at any point is due to the plume itself plus what is reflected upward from the ground.

The concentrations due to the mirror-image plume are exactly the same as those shown by equation (3.9.10), except that $(z - H)^2$ is replaced by $(z + H)^2$.i.e

$$C(y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} e^{\left(-\frac{y^2}{2\sigma^2 y} - \frac{(z+H)^2}{2\sigma^2 z}\right)} \tag{3.10.1}$$

The combined contribution of both plumes is obtained by writting equation (3.9.10) and (3.9.12) by adding the values for the two plumes and factoring out the common terms to obtain

$$C(y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} e^{\left(-\frac{y^2}{2\sigma^2 y} - \frac{(z-H)^2}{2\sigma^2 z}\right)} + \frac{Q}{2\pi u \sigma_y \sigma_z} e^{\left(-\frac{y^2}{2\sigma^2 y} - \frac{(z+H)^2}{2\sigma^2 z}\right)} \tag{3.10.2}$$

$$C = \frac{Q}{2\pi u \sigma_y \sigma_z} exp - 0.5(\frac{y}{\sigma_y})^2 [exp - 0.5(\frac{z-H}{\sigma_z})^2 + exp - 0.5(\frac{z+H}{\sigma_z})^2] \tag{3.10.3}$$

Equation (3.9.14) is correct for ground level or any elevation above it and is called Modified Gaussian Plume Equation. For large values of z, the contribution of the $(z+H)^2$ term becomes negligible and the result is practically identical with that from equation (3.9.10).

3.10.2 Mixing height limits, one-dimensional spreading

As the plume flows downwind, it will eventually grow until it is completely mixed below the mixing height H. After that it will no longer spread vertically, but only horizontally, so a two-dimensional spreading plume has converted to a one-dimensional spreading plume. The mixing height is called L, and lines are drawn for long transport distances, indicating that the observed concentrations are higher than one would compute by continuing the two-dimensional spreading calculation to those distances. Observe that H and L appear with two sets of meanings in these equations. In box models, H is the mixing height and L is the downwind length of the city. In Gaussian plume models, H is effective stack height and L is the mixing height. Alas, this usage is common. Returning to equation (3.8.28), we see that the amount being dispersed horizontally is

$$P = \frac{Q}{uL} \tag{3.10.4}$$

which accounts for the fact that the X for the two-dimensional Gaussian plume is now uniformly spread over a height L. Substituting this value in Equation (3.8.28) and making the substitutions into equation (3.9.4-3.9.6), yields to

$$c = \frac{Q}{\sqrt{2\pi}uL\sigma_y}exp^{(}-\frac{y^2}{2\sigma^2y}) \tag{3.10.5}$$

3.11 Plume rise

The visible plumes from power plants, factories and smokestacks tend to rise and then become horizontal. Plumes rise buoyantly because they are hotter than the nearby air and also since they leave the stack with a vertical velocity that transports them upward. They stop rising because, as they combine with the nearby air, they lose velocity and cool by mixing. Lastly, they level off when they come to the similar temperature as the atmosphere.

we employ plume rise calculations to estimate the value of Δh and hence of H to use in Gaussian Plume and other more complex pollutant concentration calculations. Holland's formula for plume rise is (Briggs, 1975):

$$\Delta h = \frac{V_sD}{u}(1.5 + 2.68x10^{-3}PD\frac{T_s - T_a}{T_s}) \tag{3.11.1}$$

where

Δh = plume rise in m

V_s = stack exit velocity in m/s

T_s = Stack gas temperature in K

T_a = Air temperature at stack height in K

$D =$ The diameter of the stack in m

$u =$ Wind speed in m/s

$P =$ Pressure in millibars

Equation (3.11.1) is a dimensional equation, which is only correct for the dimensions shown. This formula is frequently corrected for atmospheric stability by multiplying the result by 1.1 or 1.2 for A and B stability or 0.8 or 0.9 for D, E, or F stability. Although this formula has some theoretical basis, it is not universally applicable. All plume rise formulas work well for some cases, but none seems to handle all cases.

The next chapter will be on the methods that are used in solving the developed model equation 3.9.10.

CHAPTER 4

RESEARCH METHODOLOGY

The procedures used to attain the results documented in this thesis are deliberated in the subsequent sections.

4.1 Modified Gaussian Plume Equation.

Modified Gaussian Plume Equations is our model equation for calculating ground level concentration. It is based on known emission rates and meteorology.

$$C = \frac{Q}{2\pi u\sigma_y\sigma_z} exp - 0.5(\frac{y}{\sigma_y})^2[exp - 0.5(\frac{z-H}{\sigma_z})^2 + exp - 0.5(\frac{z+H}{\sigma_z})^2] \tag{4.1.1}$$

where H is the total elevation of the plume. This is a combination of both the physical stack height and plume rise due to buoyancy.

4.2 Atmospheric dispersion coefficients

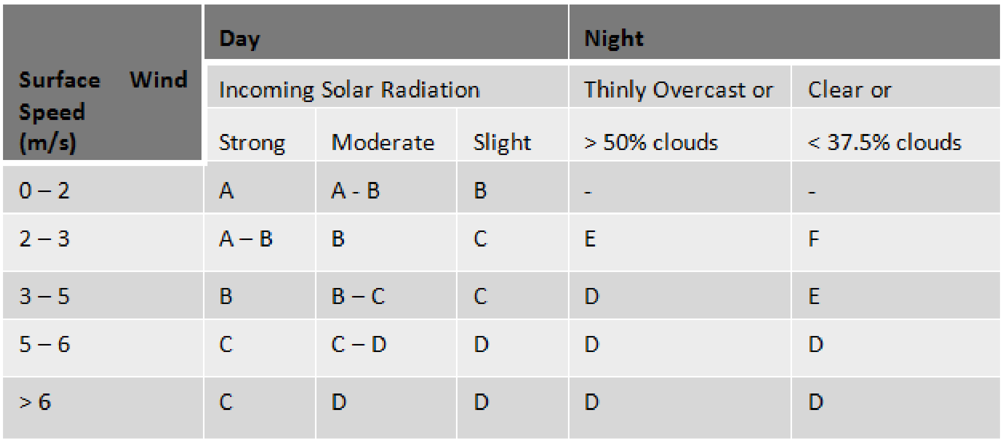
The variables σ_y and σ_z are atmospheric dispersion coefficients that have been determined empirically and are determined from the following equations:

$$\sigma_y = ax^{0.894} \tag{4.2.1}$$

$$\sigma_z = cx^d + f \tag{4.2.2}$$

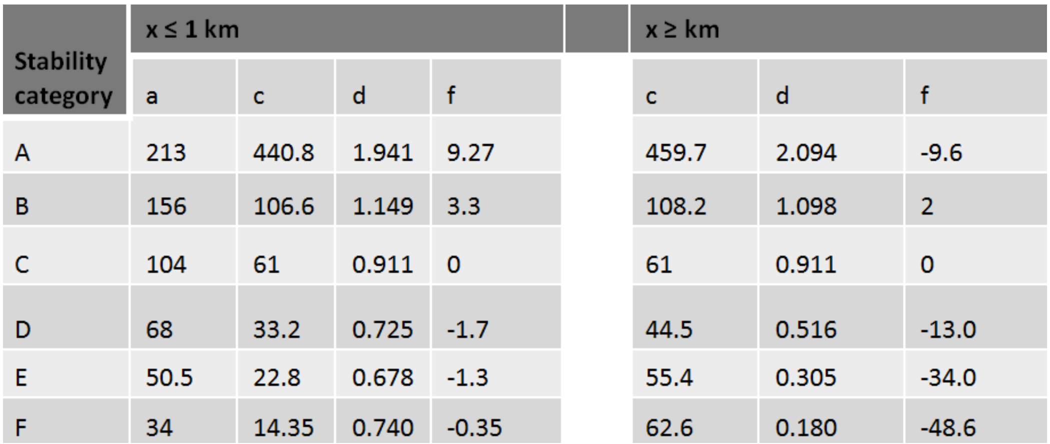
where a, c, d and f are constants given in table 4.1. These constants are functions of the atmospheric stability category for the particular site. Stability class is a function of prevailing wind speed and incoming solar radiation and can be obtained from table 4.2.

Table 4.1: shows how the atmospheric stability categories are chosen based on meteorological conditions (Zoras et al., 2006)



Surface Wind Speed (m/s)	**Day**			**Night**	
	Incoming Solar Radiation			Thinly Overcast or	Clear or
	Strong	Moderate	Slight	> 50% clouds	< 37.5% clouds
0 – 2	A	A - B	B	-	-
2 – 3	A – B	B	C	E	F
3 – 5	B	B – C	C	D	E
5 – 6	C	C – D	D	D	D
> 6	C	D	D	D	D

Table 4.2: shows how the Dispersion Coefficients are chosen based on meteorological conditions (Zoras et al., 2006).



Stability category	**x ≤ 1 km**					**x ≥ km**		
	a	c	d	f		c	d	f
A	213	440.8	1.941	9.27		459.7	2.094	-9.6
B	156	106.6	1.149	3.3		108.2	1.098	2
C	104	61	0.911	0		61	0.911	0
D	68	33.2	0.725	-1.7		44.5	0.516	-13.0
E	50.5	22.8	0.678	-1.3		55.4	0.305	-34.0
F	34	14.35	0.740	-0.35		62.6	0.180	-48.6

4.3 Plume rise

We employ plume rise calculations to estimate the value of Δh and hence of H to use in Gaussian Plume and other more complex pollutant concentration calculations. Holland's formula for plume rise is (Briggs, 1975):

$$\Delta h = \frac{V_s D}{u}(1.5 + 2.68x10^{-3} P D \frac{T_s - T_a}{T_s}) \tag{4.3.1}$$

where

Δh = plume rise in m

V_s = stack exit velocity in m/s

T_s = Stack gas temperature in K

T_a = Air temperature at stack height in K

D = The diameter of the stack in m

u = Wind speed in m/s

P = Pressure in millibars

4.4 Atmospheric mixing height.

As the plume flows downwind and expands, it reaches the atmospheric mixing height above which the plume does not rise. Dispersion cannot continue above this height and the pollutant eventually becomes completely mixed in the vertical direction. The problem now becomes a one dimensional problem and the concentrations will be greater than predicted by the $2D$ model. The mixing height equation is of the form:

$$c = \frac{Q}{\sqrt{2\pi} u L \sigma_y} e^{(-\frac{y^2}{2\sigma_y^2})} \tag{4.4.1}$$

where

Q = Emission rate

L is the mixing height

u is wind speed

In Gaussian plume model, H is the effective stack height and L is the mixing height.

4.5 Reactive or non-conservative pollutants.

If the pollutant is reactive or non-conservative (i.e. decays or changes chemical form overtime), a decay coefficient must be added to account for the decrease in pollutant mass. The decay constant is a function of the pollutants half-life:

$$K = \frac{0.693}{t^{0.5}} \tag{4.5.1}$$

From this, a reduction factor can be determined by which the Modified Gaussian Plume equation is multiplied:

$$\text{Reduction Factor} = e^{\left[\frac{-0.693x}{ut^{0.5}}\right]}$$

4.6 Air quality standard

For this thesis, the European and/ or U.S standards, have been employed for control of air quality maximum concentration limits given as follow:

Table 4.3: Air Quality maximum concentration limits

Acute Level	Chronic Level
$SO_2 : 350\mu g/m^3$ (1 hour average)	$125\mu g/m^3$ (24 hour average)
$NO_x : 200\mu g/m^3$ (1 hour average)	$40\mu g/m^3$ (1 year average)

Ground level concentrations cannot exceed the 1 hour averages anywhere downwind of the plant at any time during the year.

CHAPTER 5

MODELLING AND DATA ANALYSIS

5.1 Modelling

The overall modelling is performed as follows:

1. Determine the effective plume height based on Hollands Equation. It requires smoke stack diameter, velocity and temperature.
2. Determine the atmospheric stability category. Requires prevailing wind speed and solar radiation data.
3. Calculate σ_y and σ_z, dispersion coefficients, for increments of distance x, downwind of smoke stack.
4. Use the modified Gaussian Plume equation to calculate concentrations downwind at the ground along the plume center line. Requires pollutant, Q in grams/seconds for each pollutant.
5. Use the mixing Height Equation to calculate concentrations at the same points as in step 4. Requires estimate of mixing height, L.
6. Compare concentrations from step (4) and (5) and select the largest as the controlling concentration. Compare them to air pollution control standards.
7. The model will need to be run for different wind speeds, wind directions, and pollutant loadings (function of power output) to determine potential impact on surrounding environment.
8. To validate and calibrate the model, would need to compare output with actual field data.

5.2 Data

The data used was gathered from the Namibia meteorological weather service in Windhoek and Anixas diesel-fuelled power plant in Walvis Bay.

5.2.1 Meteorological data

The meteorology of an area plays a big role in defining the emission concentration in the atmosphere (Nagendra & Khare, 2002). The dispersion of pollutants in the atmosphere is caused by turbulent eddy movement (diffusion) and advection due to the movement of air crowds by the wind (Stockie, 2011). This means that wind speed, atmospheric stability and turbulent processes are the most significant meteorological factors prompting dispersion in the atmosphere. Additional vital meteorological factors to deliberate comprised of: wind direction, wind speed, temperature and relative humidity. Wind speed and wind direction are variable over time.

Table 5.1: Climatic conditions of Walvis Bay 2016

Monthly	Wind S (m/s)	Solar R	Cloud C %	Temp mean	Pressure
Jan	3.4	8	10	30	1011.8
Feb	3.3	7	13	31	1011.5
Mar	3.6	8	11	29	1012.6
April	3.5	7	7	28	1014.5
May	3.2	6	1	24	1018.1
June	3.9	6	3	23	1020
July	3.3	6	1	21	1019.5
Aug	3.4	7	1	25	1018
Sep	3.9	8	5	28	1014.9
Oct	3.1	8	11	32	1011.8
Nov	3.3	8	11	32	1011.5
Dec	3.6	8	8	32	1010.8
Averages	3.5	7.25	6.8	27.9	1014.58

5.2.2 Wind climate

Wind speed and direction data from Namibian Weather Service in Windhoek is used in figure 5.1. Windhoek weather station was chosen, because it is where all the data is stored in Namibia.

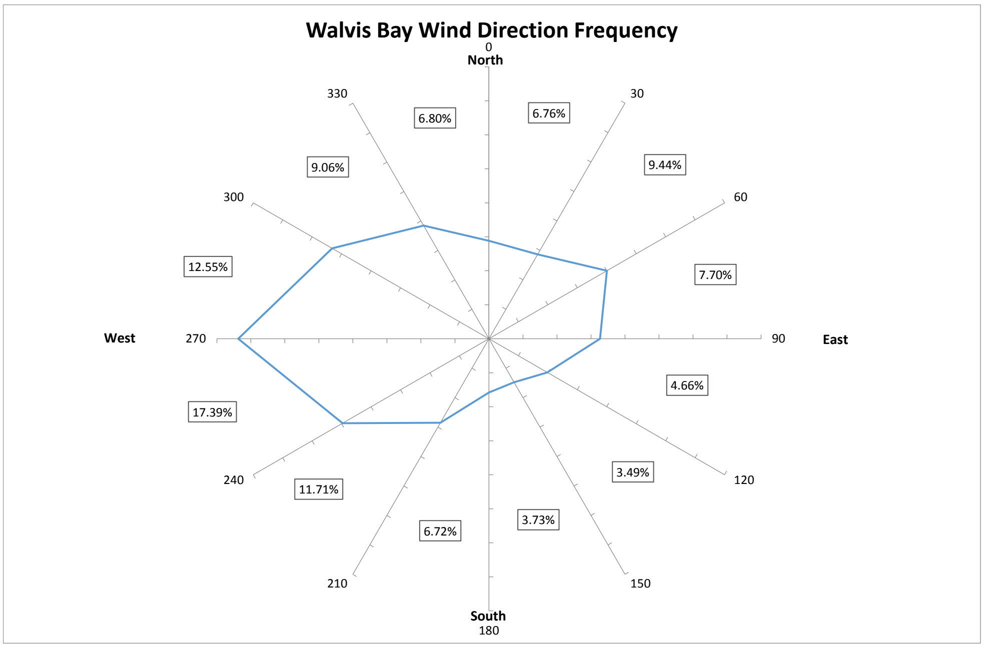


Figure 5.1: Day wind direction for Walvis Bay for 2014.

5.2.3 Temperature

As shown in figure 5.2, maximum temperatures were experienced during March and April whereas a minimum temperature was experienced during August.

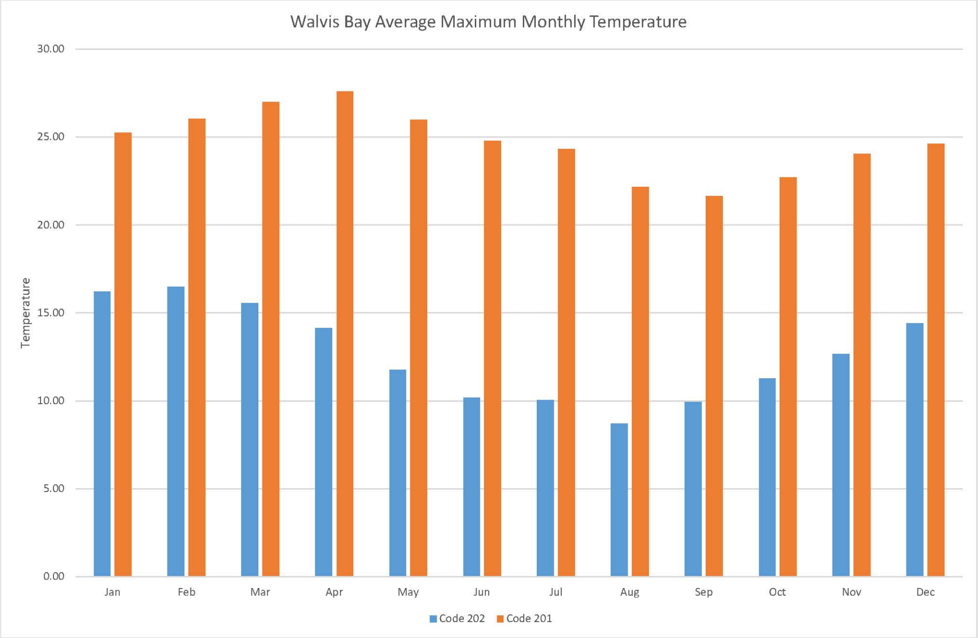


Figure 5.2: Monthly temperature profile for Walvis Bay, 2014.

5.2.4 Relative humidity

Hourly data from Windhoek Weather Station was used to plot the frequency distribution of relative humidity in certain ranges for different times of the day. Figure 5.3 shows the frequency distribution of relative humidity at 08:00, 14:00 and 20:00.

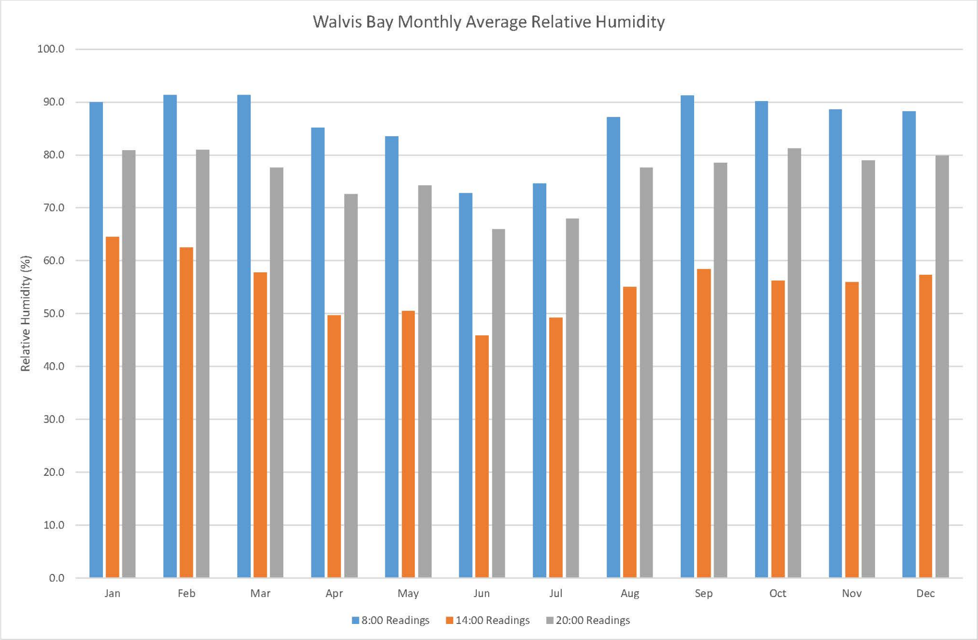


Figure 5.3: Relative humidity for Walvis Bay, 2014.

At 08:00, most hours in the period of 2014 had relative humidity values of between 73% and 91%, and at 14:00 most hours had relative humidity values between 45% and 68% while at 20:00 showed most hours had relative humidity values ranging between 65% and 81%.

5.2.5 Data about the Power Plant.

The results can only be as valid and good as input data. The following data was obtained from Anixas diesel-fuelled power station:

Table 5.2: Power plant specifications and descriptions

Stack height (H)	60 m
Stack gas temperature (T_s)	280 C/ 553.15 K
Air temp at Stack Height (T_a)	27.9 C/ 301.05K
Stack gas velocity (V_s)	15 m/s
Stack diameter (D)	3 m
Pressure (P)	1014.58 mp

5.2.6 Data from the emissions measurement system.

Minimum, maximum and averages generated by the system for the 15 - 22 August and 18 November 2013 period are given in Table 4.1 and 4.2 respectively.

Table 5.3: Average particulate and gaseous emissions for 15 - 22 August 2013.

Generator	$SO_2 (mg/m^3)$	$NO_x (mg/m^3)$	PM10 (%)
GEN1	152.1	122.3	5.4
GEN2	118.2	129.5	6.1
GEN3	5.9	9.8	5.4
AVERAGE	92.1	87.2	5.6

Table 5.4: Average particulate and gaseous emissions for 18 November 2013.

Generator	$SO_2 (mg/m^3)$	$NO_x (mg/m^3)$	PM10 (%)
GEN1	1730.6	1384.2	5.6
GEN2	1604.4	1548	52.5
GEN3	1603.2	1410	6.0
AVERAGE	1646.1	1447.4	21.4

CHAPTER 6

RESULTS, DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

6.1 Results and Discussions

Meteorology offers information of various variables that are applicable to the transport and diffusion of pollutants in the atmosphere. This section presents the outcomes founded on the previously stated approaches. It emphasises not only on wind climate, but also on further factors which have an impact on defining the air pollution climate of an area. These variables comprise wind speed, stability, mixing height, temperature and relative humidity.

The computation of concentrations and other necessary quantities have been carried out by means of excel. The resulting quantities have been plotted, as seen from figures that will follow. The raw results without visual effects are tabulated throughout Appendix A.

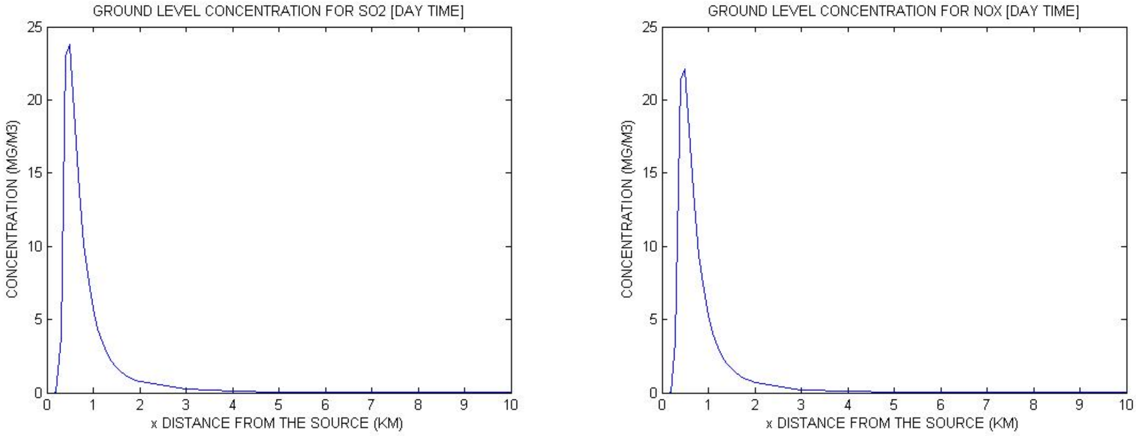


Figure 6.1: Comparison of ground level concentration for SO_2 and NO_X using stability category A

The comparison between those two gases indicated that when the wind speed is from $0 - 3$ m/s for strong to moderate incoming solar radiation, the maximum ground level concentration was $23.81\mu g/m^3$ for SO_2 and $22.11\mu g/m^3$ for NO_x at a distance of 0.5

km downstream from the power plant. Considering the impact of the mixing height, the maximum concentrations are expected to vary significantly, $9.26\mu g/m^3$ for SO_2 at a distance of 0.7 km and $14.17\mu g/m^3$ for NO_x at a distance of 0.4 km. For infinite mixing height, the maximum concentrations decreases to $4.63\mu g/m^3$ for SO_2 at a distance of 0.7 km and $7.09\mu g/m^3$ for NO_x at a distance of 0.4 km. Both of the pollutants are within the regulatory limit for air quality standard outlined for both chronic and acute limits.

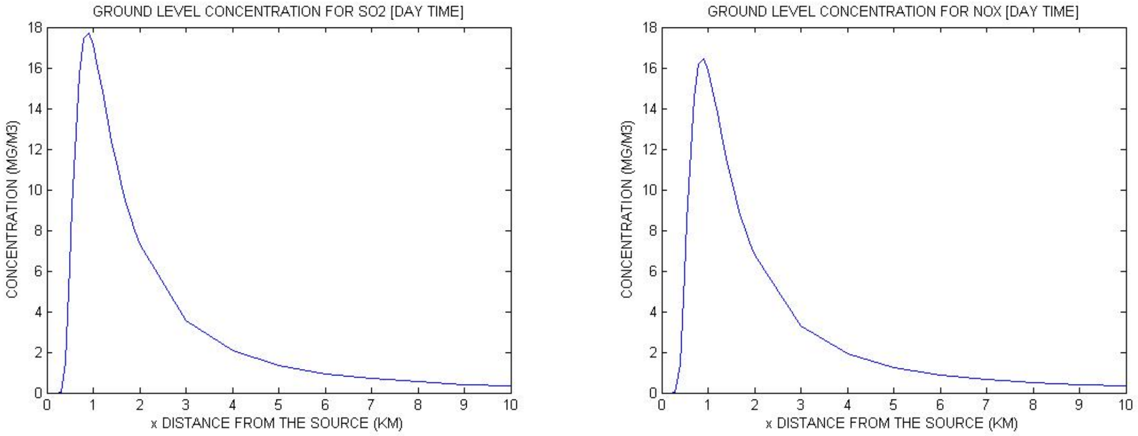


Figure 6.2: Comparison of ground level concentration for SO_2 and NO_X using stability category B

The comparison between those two gases indicated that when the wind speed is from 0−5 m/s for strong, moderate to slight incoming solar radiation, the maximum ground level concentration directly downwind during the day are $17.72\mu g/m^3$ for SO_2 and $16.45\mu g/m^3$ for NO_x at a distance of 0.9 km downstream from the power plant. Considering the impact of the mixing height, the maximum concentrations are expected to vary significantly, $12.64\mu g/m^3$ for SO_2 at a distance of 0.7 km and $19.35\mu g/m^3$ for NO_x at a distance of 0.4 km. Assuming an infinite mixing height, the maximum concentrations decreases to $6.32\mu g/m^3$ for SO_2 at a distance of 0.7 km and $9.68\mu g/m^3$ for NO_x at a distance of 0.4 km. Both pollutants are within the regulatory limit for air quality standard outlined for both chronic and acute limits.

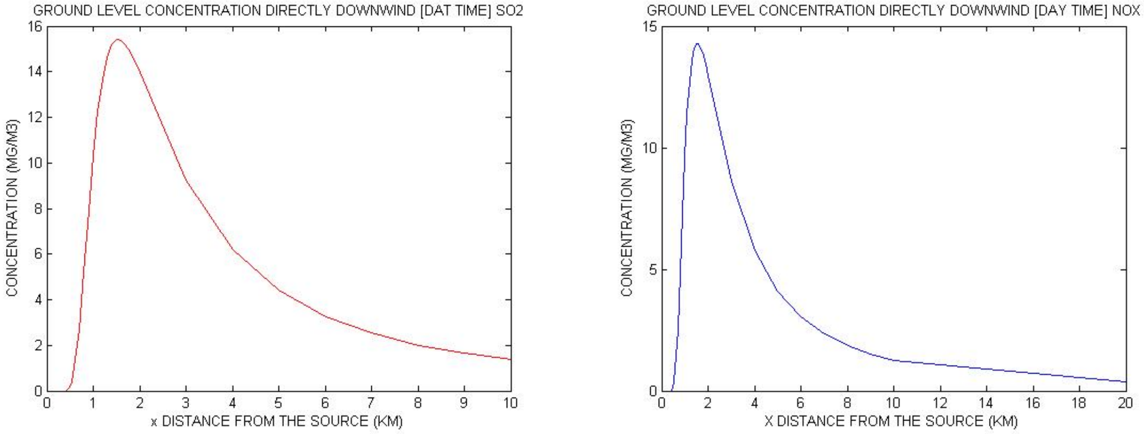


Figure 6.3: Ground level concentration directly downwind using stability category C [SO_2 and NO_X]

The comparison between those two gases indicated that when the wind speed is from 2−6 m/s for strong, moderate to slight incoming solar radiation, the maximum ground level concentration directly downwind during the day are 15.4$\mu g/m^3$ for SO_2 and 14.29$\mu g/m^3$ for NO_x at a distance of 1.5 km downstream from the power plant. Considering the impact of the mixing height, the maximum concentrations are expected to vary significantly during the day, with 18.96$\mu g/m^3$ for SO_2 at a distance of 0.7 km and 29.03$\mu g/m^3$ for NO_x at a distance of 0.4 km from the power plant. Now, assuming an infinite mixing height, the maximum concentrations decrease to 9.48$\mu g/m^3$ for SO_2 at a distance of 0.7 km and 14.51$\mu g/m^3$ for NO_x at a distance of 0.4 km. Both of the pollutants are still within the regulatory limit for air quality standard outlined for both chronic and acute limits.

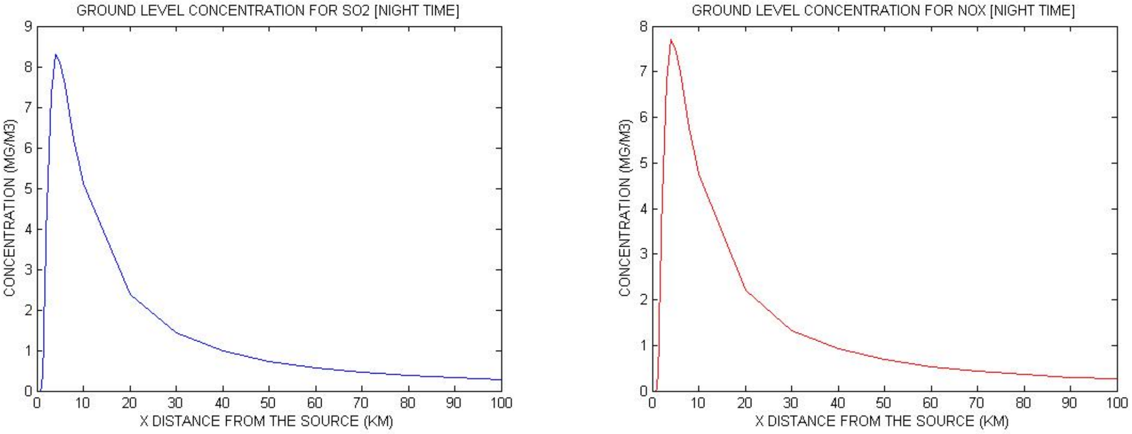


Figure 6.4: Ground level concentration directly downwind using stability category D [SO_2 and NO_X]

The comparison between those two gases indicated that when the wind speed is $3 - 6$ m/s for moderate to slight incoming solar radiation and the ground level concentration are expected to decrease during the night, with 8.29$\mu g/m^3$ for SO_2 and 7.70$\mu g/m^3$ for NO_x at a distance of 4 km downstream from the power plant, taking into consideration the prevailing wind direction with the wind speed of 3.5 m/s. This is due to the climatic conditions that are prevalent during the night such as fog and cloud cover resulting in concentration build up.

However, considering the impact of the mixing height, the maximum concentrations are expected to vary significantly during the night, with 28.99$\mu g/m^3$ for SO_2 and 29.03$\mu g/m^3$ for NO_x at a distance of 0.7 km and 0.4 km respectively. Now, assuming an infinite mixing height, the maximum concentrations decrease to 14.5$\mu g/m^3$ for SO_2 at a distance of 0.7 km and 22.2$\mu g/m^3$ for NO_x at a distance of 0.4 km. Hence, both pollutants are still within the regulatory limit for air quality standard outlined for both chronic and acute limits.

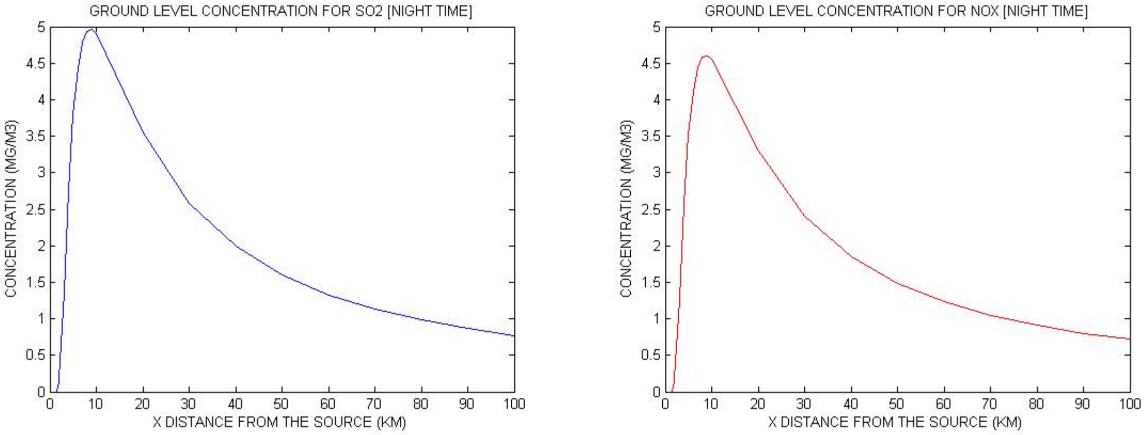


Figure 6.5: Comparison of ground level concentration for SO_2 and NO_X using stability category E

The comparison between those two gases indicated that when the wind speed is $2-5$ m/s for thinly overcast or greater than 50 % clouds to clear or less than 37.5% clouds, the ground level concentration are expected to very low during the night, with $4.96\mu g/m^3$ for SO_2 and $4.60\mu g/m^3$ for NO_x at a distance of 9 km downstream from the power plant, taking into consideration the prevailing wind direction. This is due to the climatic conditions that are prevalent during the night such as fog and cloud cover resulting in concentration build up.

However, considering the impact of the mixing height, the maximum concentrations are expected to vary significantly during the night, with $39.04\mu g/m^3$ for SO_2 and $59.78\mu g/m^3$ for NO_x at a distance of 700 m and 400 m respectively. Now, assuming an infinite mixing height, the maximum concentrations decrease to $19.52\mu g/m^3$ for SO_2 at a distance of 0.7 km and $29.89\mu g/m^3$ for NO_x at a distance of 0.4 km. Hence, both of our pollutants are still within the regulatory limit for air quality standard outlined for both chronic and acute limits.

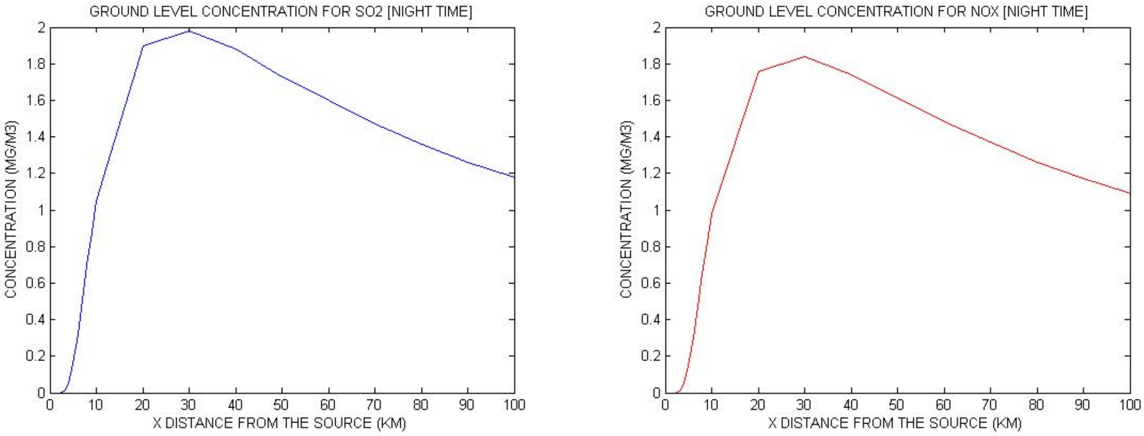


Figure 6.6: Comparison of ground level concentration for SO_2 and NO_X using stability category F

The comparison between those two gases indicated that when the wind speed is $2-3$ m/s for clear or less than 37.5% clouds, the ground level concentrations are expected to be very high during the night, with $1.98\mu g/m^3$ for SO_2 and $1.84\mu g/m^3$ for NO_x at the distance of 10 km downstream from the power plant, taking into consideration the prevailing wind direction. This is due to the climatic conditions that are prevalent during the night such as fog and cloud cover resulting in concentration build up.

However, considering the impact of the mixing height, the maximum concentrations are expected to vary significantly during the night, with $57.99\mu g/m^3$ for SO_2 and $88.79\mu g/m^3$ for NO_x at a distance of 700 m and 400 m respectively. Now, assuming an infinite mixing height, the maximum concentrations decrease to $28.99\mu g/m^3$ for SO_2 at a distance of 0.7 km and $44.39\mu g/m^3$ for NO_x at a distance of 0.4 km. Hence, both of our pollutants are still within the regulatory limit for air quality standard outlined for both chronic and acute limits.

Nonetheless, regardless of the climatic conditions and direction of the prevailing wind directions, installing a stack height of +50 m for the power plant and the current moderate average wind speed, the observed annual maximum ground level concentrations are expected to be within the regulatory limit outlined in the air quality standard for acute level of $350\mu g/m^2$ for SO_2 and $200\mu g/m^2$ for NO_x concentration for $x \geq$ 1km directly downwind and/or downstream.

6.2 Conclusions

In this study, the effect of air emission from diesel power plant using air quality was assessed. A typical case of Anixas Power Plant on air pollution impact forecasting due to SO_2 and NO_X emissions from the given stack through mathematical modelling approach was presented. The study revealed that the contribution of atmospheric contaminants of air emission due to the studied power plant (Anixas Power Plant, Walvis Bay) was very low. The study mostly concentrated only on SO_2 and NO_X since the emission of PM was very low compared to SO_2 and NO_X.

Data from the power plant measuring system indicates that the hourly averages from 07h00 - 23h00 for SO_2 and NO_X was above $1000\mu g/m^3$ in all the three generators. The results of the comparisons between the calculated and measured concentrations for the ground level measurements, both short term averages and long term averages, indicated that the model could predict ground level concentrations within the limit.

For any one stability category, raising the point of emission (increasing the value of H) lowers the calculated ground-level concentrations for all points near the stack. For points far enough away from the plume to be well mixed up to the mixing height, the calculated concentration becomes independent of the stack height. Raising the height of emissions would lower all nearby concentrations and not change the concentrations at a distance.

Analysis of the emission of the pollutant has the following parameters; wind speed, wind direction, atmospheric stability conditions, the mixing heights, temperature and relative humidity. All these parameters have an impact on atmospheric dispersion of contaminants from the given power plant in the region. For example, low wind speeds, mixing height and stable conditions will cause the conditions of small amount of emissions dispersion in the atmosphere.

Although the Gaussian plume model is based upon many simplifying assumptions about the dispersion of pollutants in the atmosphere, it is applied to a wide array of dispersion scenarios. Some form of this model is adopted in most regulatory air pollution models for continuous release. In order to extend the application of the Gaussian plume model to realistic cases, the model makes use of different mathematical equations.

Pollutant concentration models are rely on known emission rates and meteorology. These models play an important role in the Air Quality Management type of air pollution control strategy currently used in the United States and much of the rest of the world. Gaussian plume models are widely used for point sources. They depends on several simplifying assumptions but have been reasonably successful in predicting experimental results for single, elevated point sources.

The dispersion of contaminants in the atmosphere is commonly investigated in both Eulerian and Lagrangian frameworks. The Lagrangian approach considers the trajectory of marked fluid particles in the flow. Lagrangian particle models are very useful for describing the turbulent dispersion of passive contaminants because they can take into account essential aspects of turbulence, although they are limited to a simplified set of reacting species. The Eulerian approach, on the other hand, is based on the mass conservation equation and can incorporate the various second and high-order chemical kinetic equations necessary to describe photochemical smog generation, which is a challenging open problem.

6.3 Recommendations

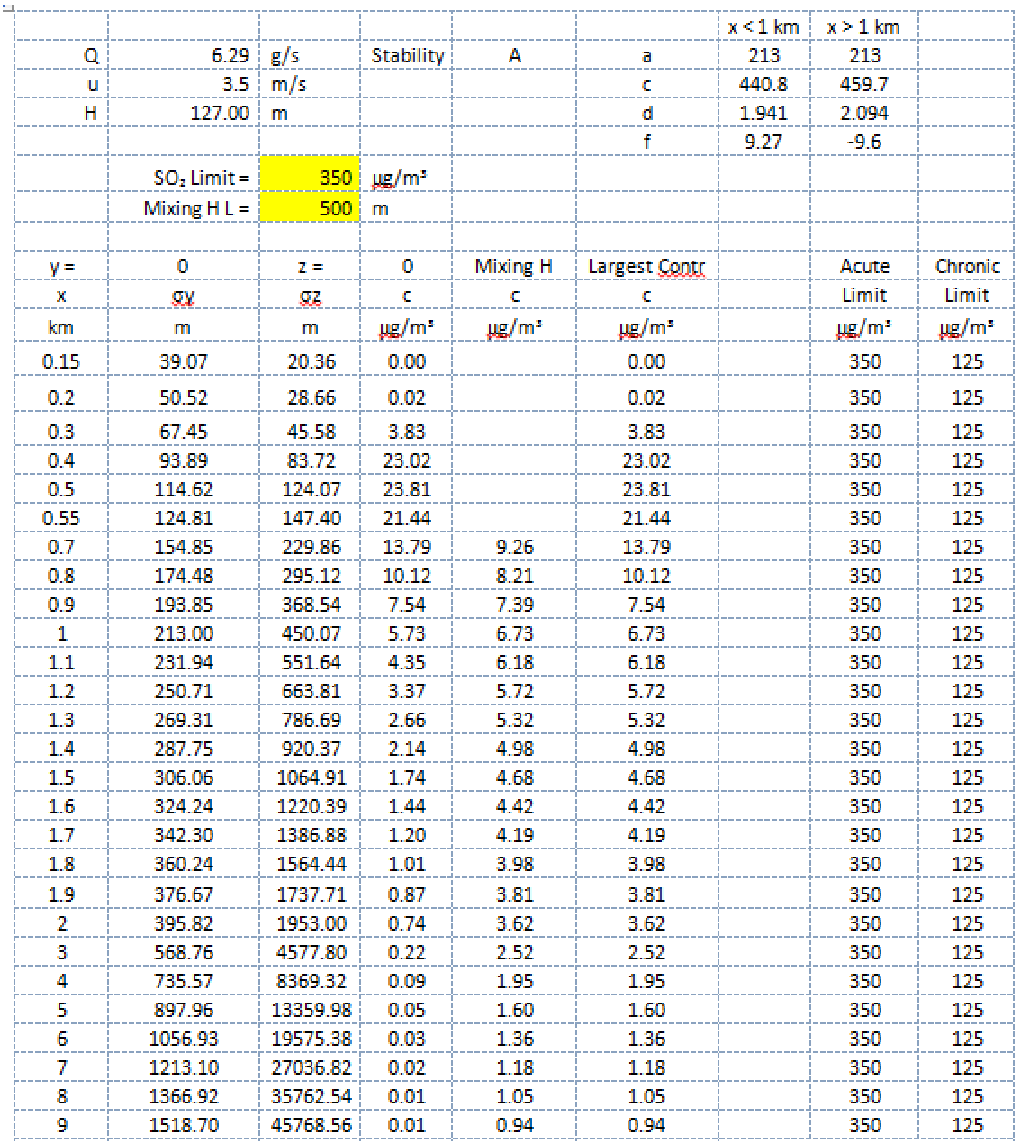
It is very difficult and expensive to relocate a plant which is in existence, but a new plant can be situated where its emissions will have their greatest impact in non-populated areas. This reasoning is the basis for most industrial zoning and land-use planning regulations. We therefore recommend that for a new power plants which is use diesel fuel, it must be placed far from people and it must have stuck height which is above 60m.

Modelling results should be carefully analysed and interpreted. The uncertainties associated with the input data, and with the model itself, need to be properly evaluated before their predictions can be used with confidence. Dispersion is primarily controlled by turbulence, which is random by nature, and thus cannot be precisely described or predicted by means of basic statistical properties. As a result, there are spatial and temporal variability that occur naturally in the observed concentration field. On the other hand, uncertainty in the model results could also be due to factors such as errors in the input data and model formulation. Because of the effects of uncertainty and its inherent randomness, it is not possible for an air quality model ever to be perfect, and there is always a base amount of scatter that cannot be removed.

APPENDIX A

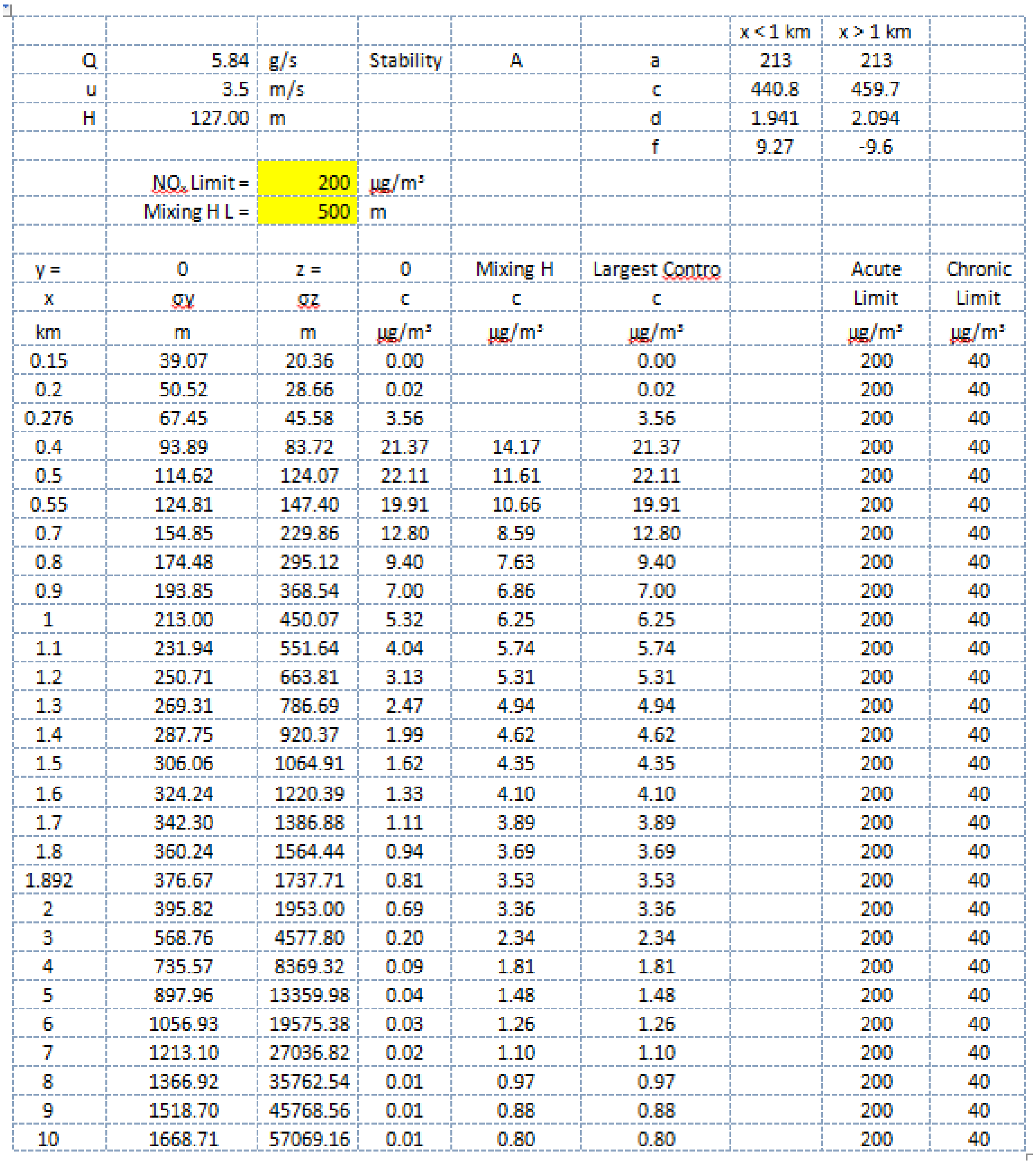
Tables

Table A.1: Truncated data for SO_2 using stability category A



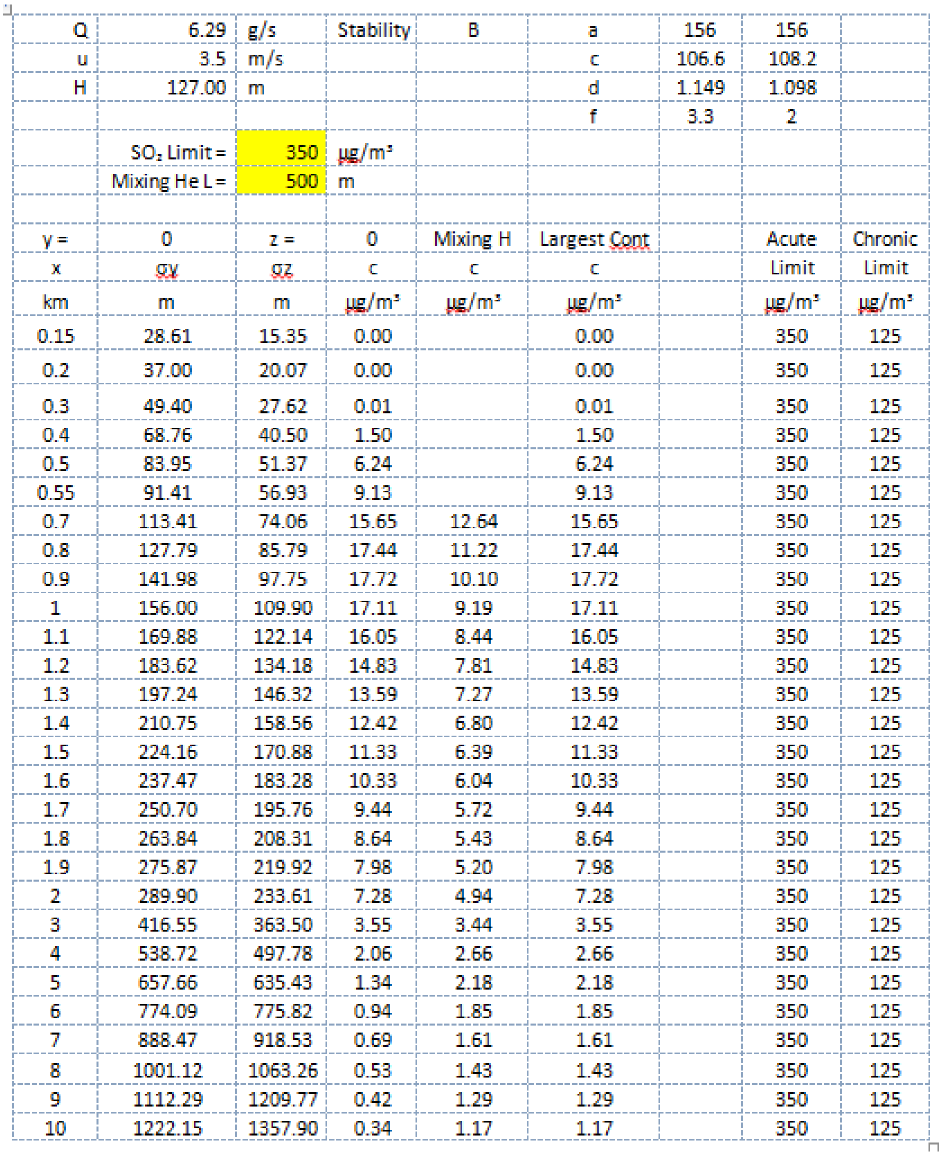
						x < 1 km	x > 1 km	
Q	6.29	g/s	Stability	A	a	213	213	
u	3.5	m/s			c	440.8	459.7	
H	127.00	m			d	1.941	2.094	
					f	9.27	-9.6	
	SO₂ Limit =	350	µg/m³					
	Mixing H L =	500	m					
y =	0	z =	0	Mixing H	Largest Contr		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	µg/m³	µg/m³	µg/m³		µg/m³	µg/m³
0.15	39.07	20.36	0.00		0.00		350	125
0.2	50.52	28.66	0.02		0.02		350	125
0.3	67.45	45.58	3.83		3.83		350	125
0.4	93.89	83.72	23.02		23.02		350	125
0.5	114.62	124.07	23.81		23.81		350	125
0.55	124.81	147.40	21.44		21.44		350	125
0.7	154.85	229.86	13.79	9.26	13.79		350	125
0.8	174.48	295.12	10.12	8.21	10.12		350	125
0.9	193.85	368.54	7.54	7.39	7.54		350	125
1	213.00	450.07	5.73	6.73	6.73		350	125
1.1	231.94	551.64	4.35	6.18	6.18		350	125
1.2	250.71	663.81	3.37	5.72	5.72		350	125
1.3	269.31	786.69	2.66	5.32	5.32		350	125
1.4	287.75	920.37	2.14	4.98	4.98		350	125
1.5	306.06	1064.91	1.74	4.68	4.68		350	125
1.6	324.24	1220.39	1.44	4.42	4.42		350	125
1.7	342.30	1386.88	1.20	4.19	4.19		350	125
1.8	360.24	1564.44	1.01	3.98	3.98		350	125
1.9	376.67	1737.71	0.87	3.81	3.81		350	125
2	395.82	1953.00	0.74	3.62	3.62		350	125
3	568.76	4577.80	0.22	2.52	2.52		350	125
4	735.57	8369.32	0.09	1.95	1.95		350	125
5	897.96	13359.98	0.05	1.60	1.60		350	125
6	1056.93	19575.38	0.03	1.36	1.36		350	125
7	1213.10	27036.82	0.02	1.18	1.18		350	125
8	1366.92	35762.54	0.01	1.05	1.05		350	125
9	1518.70	45768.56	0.01	0.94	0.94		350	125

Table A.2: Truncated data for NO_x using stability category A



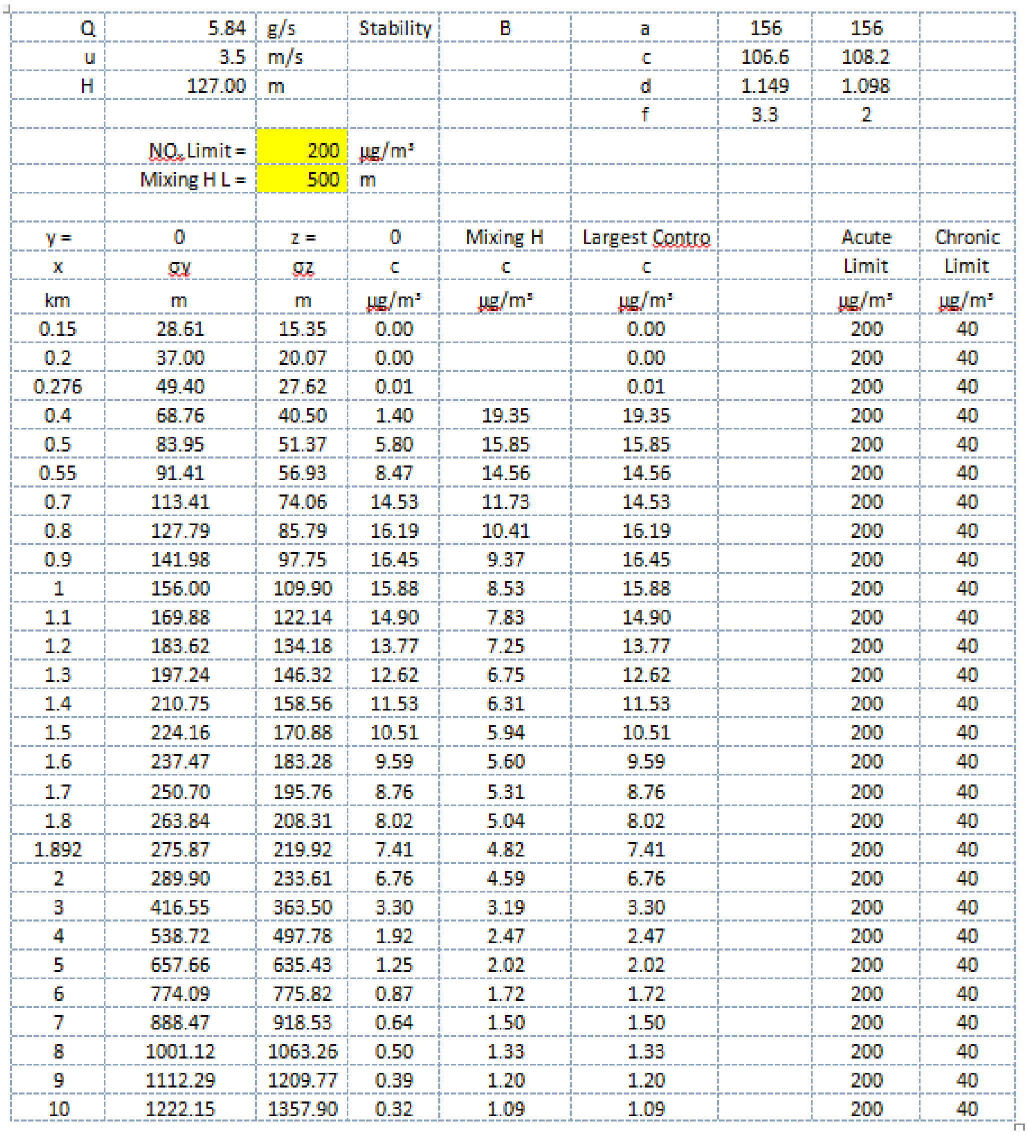
						x < 1 km	x > 1 km		
Q	5.84	g/s	Stability	A	a	213	213		
u	3.5	m/s			c	440.8	459.7		
H	127.00	m			d	1.941	2.094		
					f	9.27	-9.6		
	NO_x Limit =	200	µg/m³						
	Mixing H L =	500	m						
y =	0	z =	0	Mixing H	Largest Contro		Acute	Chronic	
x	σy	σz	c	c	c		Limit	Limit	
km	m	m	µg/m³	µg/m³	µg/m³		µg/m³	µg/m³	
0.15	39.07	20.36	0.00		0.00		200	40	
0.2	50.52	28.66	0.02		0.02		200	40	
0.276	67.45	45.58	3.56		3.56		200	40	
0.4	93.89	83.72	21.37	14.17	21.37		200	40	
0.5	114.62	124.07	22.11	11.61	22.11		200	40	
0.55	124.81	147.40	19.91	10.66	19.91		200	40	
0.7	154.85	229.86	12.80	8.59	12.80		200	40	
0.8	174.48	295.12	9.40	7.63	9.40		200	40	
0.9	193.85	368.54	7.00	6.86	7.00		200	40	
1	213.00	450.07	5.32	6.25	6.25		200	40	
1.1	231.94	551.64	4.04	5.74	5.74		200	40	
1.2	250.71	663.81	3.13	5.31	5.31		200	40	
1.3	269.31	786.69	2.47	4.94	4.94		200	40	
1.4	287.75	920.37	1.99	4.62	4.62		200	40	
1.5	306.06	1064.91	1.62	4.35	4.35		200	40	
1.6	324.24	1220.39	1.33	4.10	4.10		200	40	
1.7	342.30	1386.88	1.11	3.89	3.89		200	40	
1.8	360.24	1564.44	0.94	3.69	3.69		200	40	
1.892	376.67	1737.71	0.81	3.53	3.53		200	40	
2	395.82	1953.00	0.69	3.36	3.36		200	40	
3	568.76	4577.80	0.20	2.34	2.34		200	40	
4	735.57	8369.32	0.09	1.81	1.81		200	40	
5	897.96	13359.98	0.04	1.48	1.48		200	40	
6	1056.93	19575.38	0.03	1.26	1.26		200	40	
7	1213.10	27036.82	0.02	1.10	1.10		200	40	
8	1366.92	35762.54	0.01	0.97	0.97		200	40	
9	1518.70	45768.56	0.01	0.88	0.88		200	40	
10	1668.71	57069.16	0.01	0.80	0.80		200	40	

Table A.3: Truncated data for SO_2 using stability category B



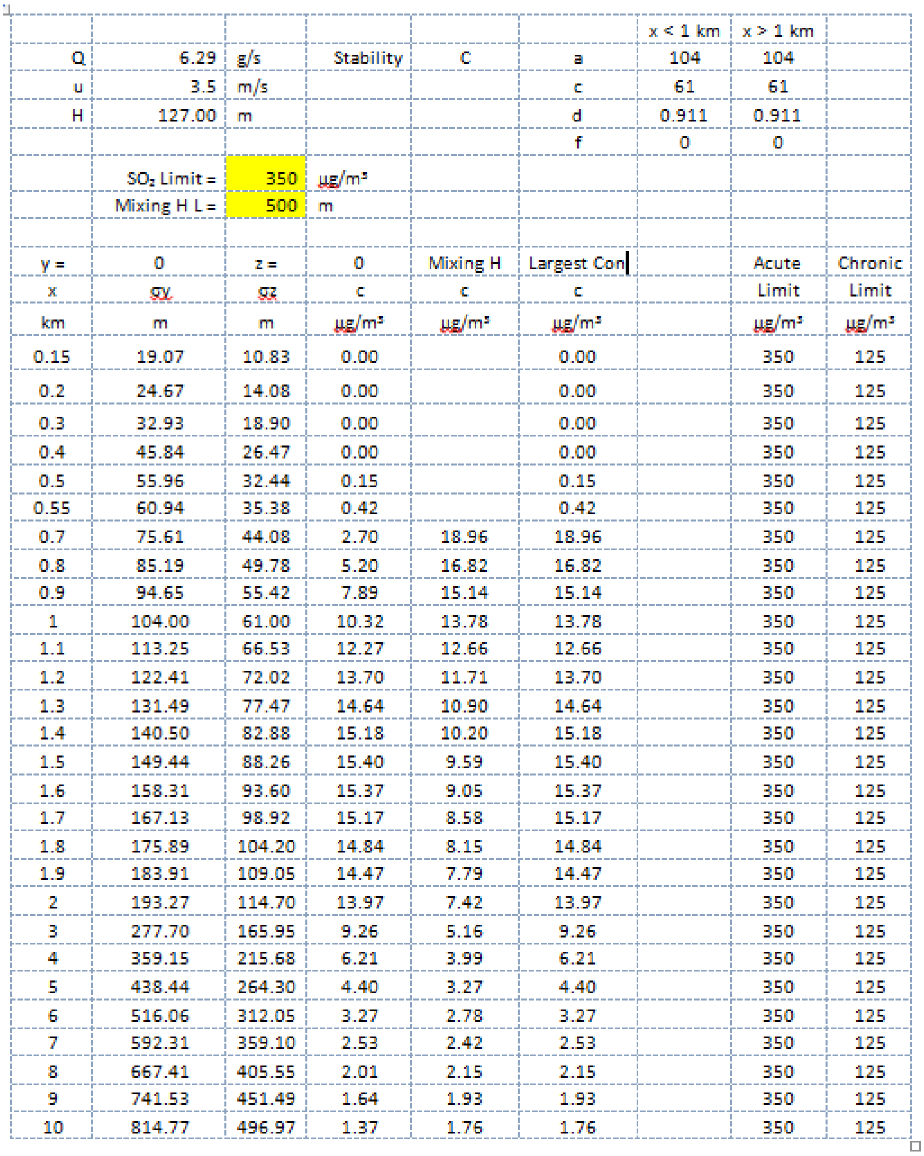
Q	6.29	g/s	Stability	B	a	156	156	
u	3.5	m/s			c	106.6	108.2	
H	127.00	m			d	1.149	1.098	
					f	3.3	2	
	SO_2 Limit =	350	µg/m³					
	Mixing He L =	500	m					
y =	0	z =	0	Mixing H	Largest Cont		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	µg/m³	µg/m³	µg/m³		µg/m³	µg/m³
0.15	28.61	15.35	0.00		0.00		350	125
0.2	37.00	20.07	0.00		0.00		350	125
0.3	49.40	27.62	0.01		0.01		350	125
0.4	68.76	40.50	1.50		1.50		350	125
0.5	83.95	51.37	6.24		6.24		350	125
0.55	91.41	56.93	9.13		9.13		350	125
0.7	113.41	74.06	15.65	12.64	15.65		350	125
0.8	127.79	85.79	17.44	11.22	17.44		350	125
0.9	141.98	97.75	17.72	10.10	17.72		350	125
1	156.00	109.90	17.11	9.19	17.11		350	125
1.1	169.88	122.14	16.05	8.44	16.05		350	125
1.2	183.62	134.18	14.83	7.81	14.83		350	125
1.3	197.24	146.32	13.59	7.27	13.59		350	125
1.4	210.75	158.56	12.42	6.80	12.42		350	125
1.5	224.16	170.88	11.33	6.39	11.33		350	125
1.6	237.47	183.28	10.33	6.04	10.33		350	125
1.7	250.70	195.76	9.44	5.72	9.44		350	125
1.8	263.84	208.31	8.64	5.43	8.64		350	125
1.9	275.87	219.92	7.98	5.20	7.98		350	125
2	289.90	233.61	7.28	4.94	7.28		350	125
3	416.55	363.50	3.55	3.44	3.55		350	125
4	538.72	497.78	2.06	2.66	2.66		350	125
5	657.66	635.43	1.34	2.18	2.18		350	125
6	774.09	775.82	0.94	1.85	1.85		350	125
7	888.47	918.53	0.69	1.61	1.61		350	125
8	1001.12	1063.26	0.53	1.43	1.43		350	125
9	1112.29	1209.77	0.42	1.29	1.29		350	125
10	1222.15	1357.90	0.34	1.17	1.17		350	125

Table A.4: Truncated data for NO_x using stability category B



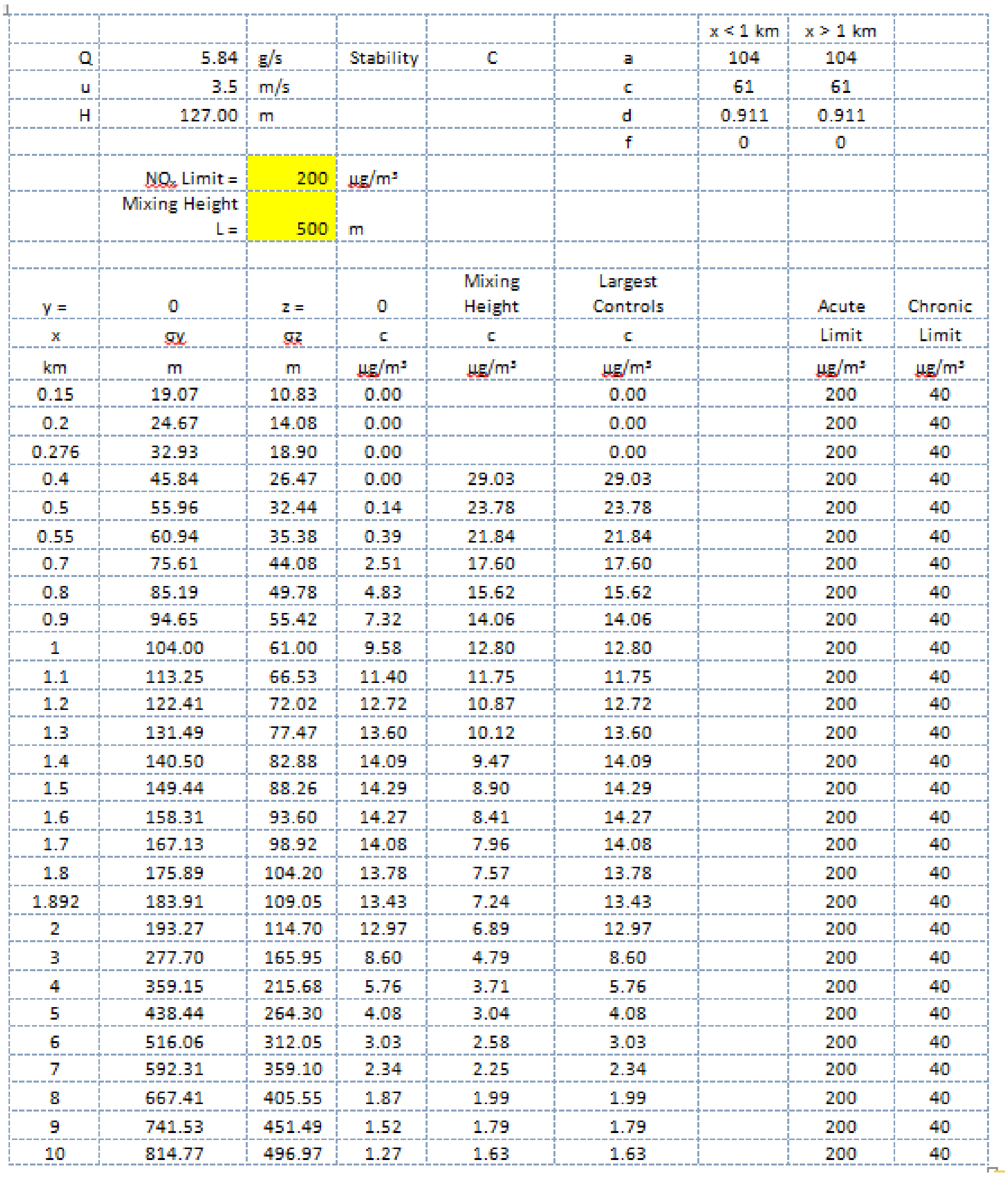
Q	5.84	g/s	Stability	B	a	156	156	
u	3.5	m/s			c	106.6	108.2	
H	127.00	m			d	1.149	1.098	
					f	3.3	2	
	NO_x Limit =	200	µg/m³					
	Mixing H L =	500	m					
y =	0	z =	0	Mixing H	Largest Contro		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	µg/m³	µg/m³	µg/m³		µg/m³	µg/m³
0.15	28.61	15.35	0.00		0.00		200	40
0.2	37.00	20.07	0.00		0.00		200	40
0.276	49.40	27.62	0.01		0.01		200	40
0.4	68.76	40.50	1.40	19.35	19.35		200	40
0.5	83.95	51.37	5.80	15.85	15.85		200	40
0.55	91.41	56.93	8.47	14.56	14.56		200	40
0.7	113.41	74.06	14.53	11.73	14.53		200	40
0.8	127.79	85.79	16.19	10.41	16.19		200	40
0.9	141.98	97.75	16.45	9.37	16.45		200	40
1	156.00	109.90	15.88	8.53	15.88		200	40
1.1	169.88	122.14	14.90	7.83	14.90		200	40
1.2	183.62	134.18	13.77	7.25	13.77		200	40
1.3	197.24	146.32	12.62	6.75	12.62		200	40
1.4	210.75	158.56	11.53	6.31	11.53		200	40
1.5	224.16	170.88	10.51	5.94	10.51		200	40
1.6	237.47	183.28	9.59	5.60	9.59		200	40
1.7	250.70	195.76	8.76	5.31	8.76		200	40
1.8	263.84	208.31	8.02	5.04	8.02		200	40
1.892	275.87	219.92	7.41	4.82	7.41		200	40
2	289.90	233.61	6.76	4.59	6.76		200	40
3	416.55	363.50	3.30	3.19	3.30		200	40
4	538.72	497.78	1.92	2.47	2.47		200	40
5	657.66	635.43	1.25	2.02	2.02		200	40
6	774.09	775.82	0.87	1.72	1.72		200	40
7	888.47	918.53	0.64	1.50	1.50		200	40
8	1001.12	1063.26	0.50	1.33	1.33		200	40
9	1112.29	1209.77	0.39	1.20	1.20		200	40
10	1222.15	1357.90	0.32	1.09	1.09		200	40

Table A.5: Truncated data for SO_2 using stability category C



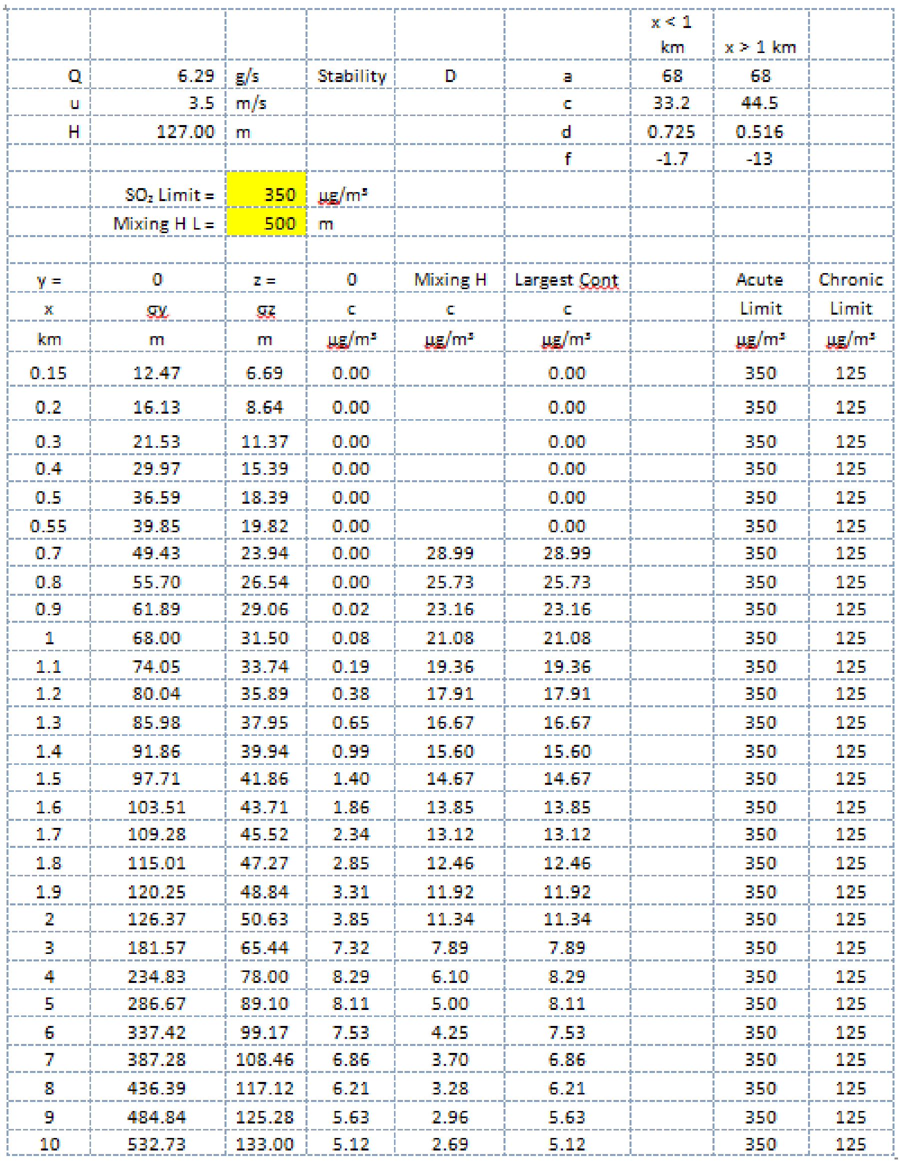
						x < 1 km	x > 1 km	
Q	6.29	g/s	Stability	C	a	104	104	
u	3.5	m/s			c	61	61	
H	127.00	m			d	0.911	0.911	
					f	0	0	
	SO_2 Limit =	350	µg/m³					
	Mixing H L =	500	m					
y =	0	z =	0	Mixing H	Largest Con		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	µg/m³	µg/m³	µg/m³		µg/m³	µg/m³
0.15	19.07	10.83	0.00		0.00		350	125
0.2	24.67	14.08	0.00		0.00		350	125
0.3	32.93	18.90	0.00		0.00		350	125
0.4	45.84	26.47	0.00		0.00		350	125
0.5	55.96	32.44	0.15		0.15		350	125
0.55	60.94	35.38	0.42		0.42		350	125
0.7	75.61	44.08	2.70	18.96	18.96		350	125
0.8	85.19	49.78	5.20	16.82	16.82		350	125
0.9	94.65	55.42	7.89	15.14	15.14		350	125
1	104.00	61.00	10.32	13.78	13.78		350	125
1.1	113.25	66.53	12.27	12.66	12.66		350	125
1.2	122.41	72.02	13.70	11.71	13.70		350	125
1.3	131.49	77.47	14.64	10.90	14.64		350	125
1.4	140.50	82.88	15.18	10.20	15.18		350	125
1.5	149.44	88.26	15.40	9.59	15.40		350	125
1.6	158.31	93.60	15.37	9.05	15.37		350	125
1.7	167.13	98.92	15.17	8.58	15.17		350	125
1.8	175.89	104.20	14.84	8.15	14.84		350	125
1.9	183.91	109.05	14.47	7.79	14.47		350	125
2	193.27	114.70	13.97	7.42	13.97		350	125
3	277.70	165.95	9.26	5.16	9.26		350	125
4	359.15	215.68	6.21	3.99	6.21		350	125
5	438.44	264.30	4.40	3.27	4.40		350	125
6	516.06	312.05	3.27	2.78	3.27		350	125
7	592.31	359.10	2.53	2.42	2.53		350	125
8	667.41	405.55	2.01	2.15	2.15		350	125
9	741.53	451.49	1.64	1.93	1.93		350	125
10	814.77	496.97	1.37	1.76	1.76		350	125

Table A.6: Truncated data for NO_x using stability category C



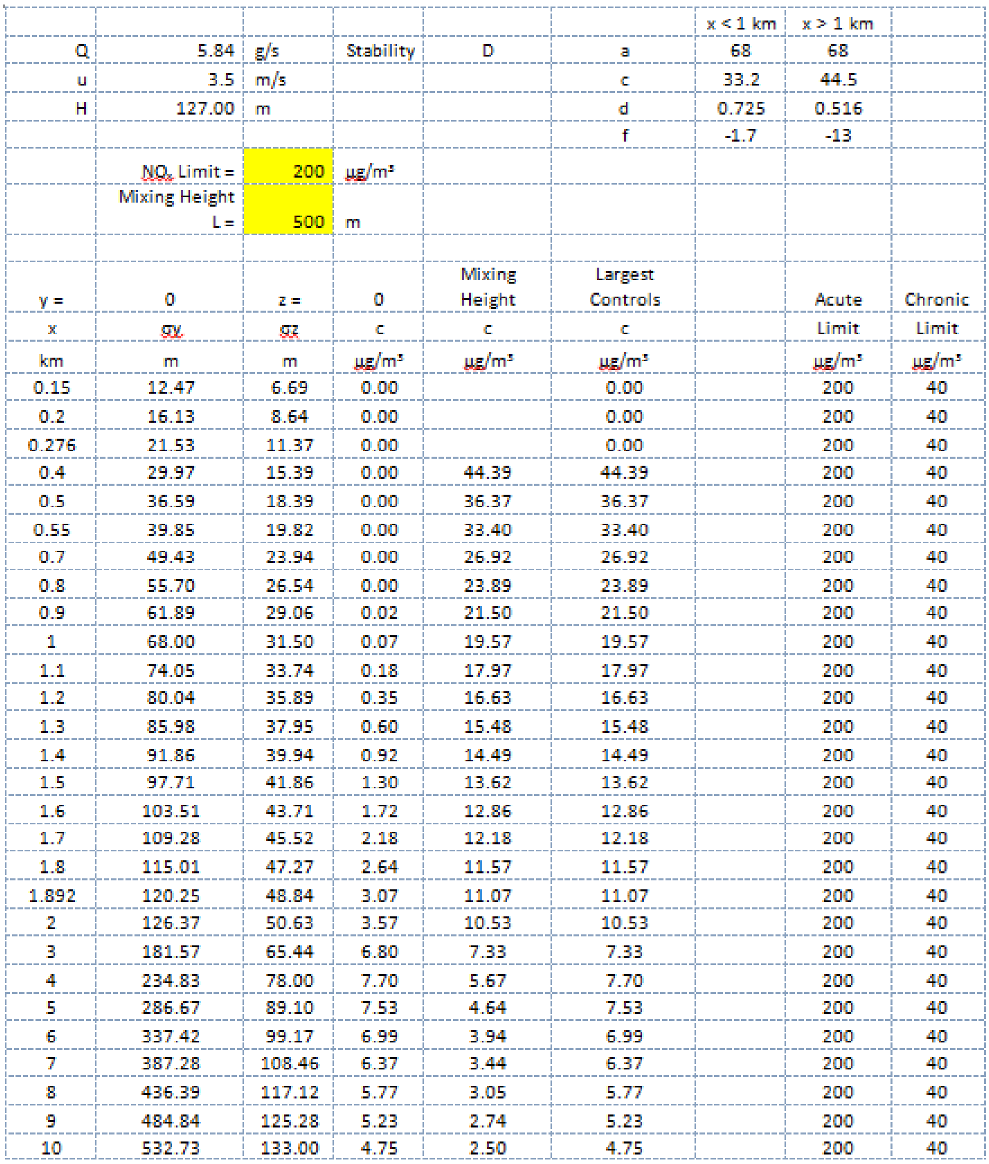
						x < 1 km	x > 1 km	
Q	5.84	g/s	Stability	C	a	104	104	
u	3.5	m/s			c	61	61	
H	127.00	m			d	0.911	0.911	
					f	0	0	
	NO_x Limit =	200	µg/m³					
	Mixing Height L =	500	m					
y =	0	z =	0	Mixing Height	Largest Controls		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	µg/m³	µg/m³	µg/m³		µg/m³	µg/m³
0.15	19.07	10.83	0.00		0.00		200	40
0.2	24.67	14.08	0.00		0.00		200	40
0.276	32.93	18.90	0.00		0.00		200	40
0.4	45.84	26.47	0.00	29.03	29.03		200	40
0.5	55.96	32.44	0.14	23.78	23.78		200	40
0.55	60.94	35.38	0.39	21.84	21.84		200	40
0.7	75.61	44.08	2.51	17.60	17.60		200	40
0.8	85.19	49.78	4.83	15.62	15.62		200	40
0.9	94.65	55.42	7.32	14.06	14.06		200	40
1	104.00	61.00	9.58	12.80	12.80		200	40
1.1	113.25	66.53	11.40	11.75	11.75		200	40
1.2	122.41	72.02	12.72	10.87	12.72		200	40
1.3	131.49	77.47	13.60	10.12	13.60		200	40
1.4	140.50	82.88	14.09	9.47	14.09		200	40
1.5	149.44	88.26	14.29	8.90	14.29		200	40
1.6	158.31	93.60	14.27	8.41	14.27		200	40
1.7	167.13	98.92	14.08	7.96	14.08		200	40
1.8	175.89	104.20	13.78	7.57	13.78		200	40
1.892	183.91	109.05	13.43	7.24	13.43		200	40
2	193.27	114.70	12.97	6.89	12.97		200	40
3	277.70	165.95	8.60	4.79	8.60		200	40
4	359.15	215.68	5.76	3.71	5.76		200	40
5	438.44	264.30	4.08	3.04	4.08		200	40
6	516.06	312.05	3.03	2.58	3.03		200	40
7	592.31	359.10	2.34	2.25	2.34		200	40
8	667.41	405.55	1.87	1.99	1.99		200	40
9	741.53	451.49	1.52	1.79	1.79		200	40
10	814.77	496.97	1.27	1.63	1.63		200	40

Table A.7: Truncated data for SO_2 using stability category D



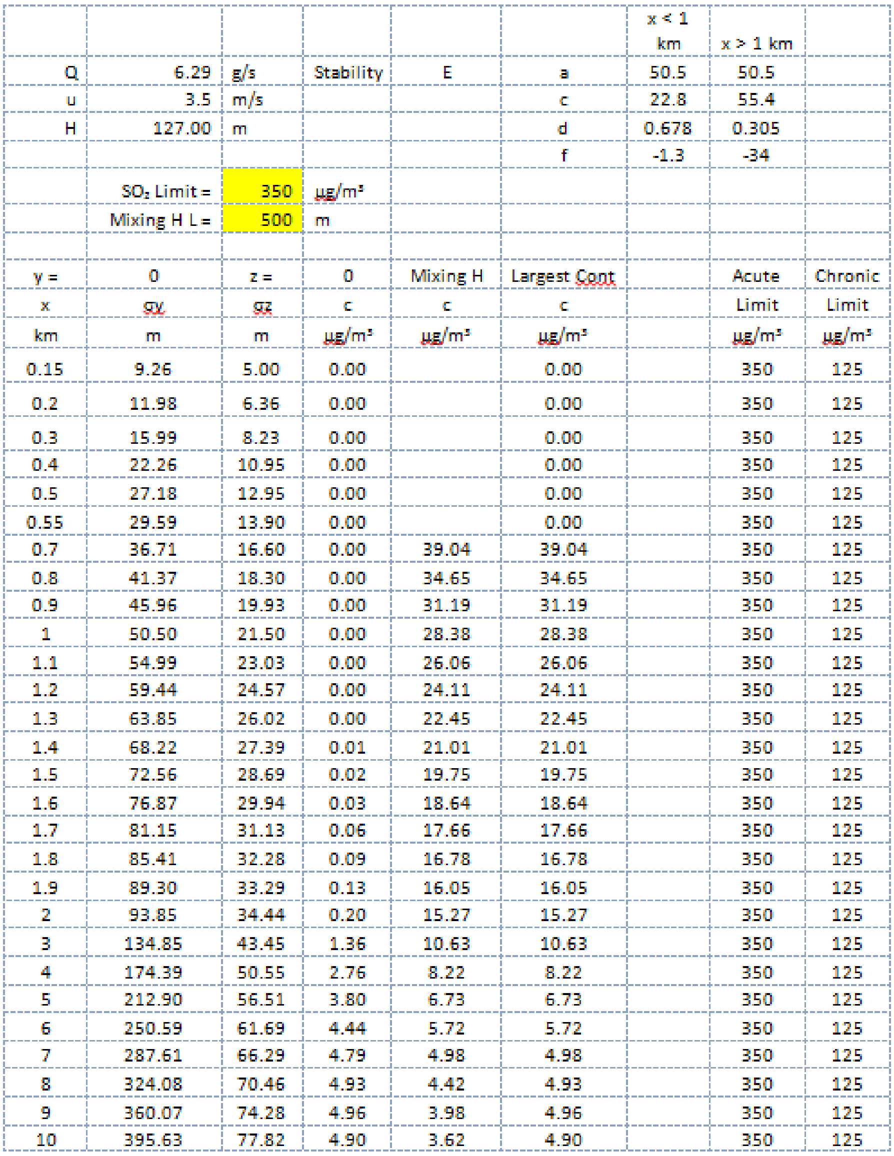
						x < 1 km	x > 1 km	
Q	6.29	g/s	Stability	D	a	68	68	
u	3.5	m/s			c	33.2	44.5	
H	127.00	m			d	0.725	0.516	
					f	-1.7	-13	
	SO_2 Limit =	350	μg/m³					
	Mixing H L =	500	m					
y =	0	z =	0	Mixing H	Largest Cont		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	μg/m³	μg/m³	μg/m³		μg/m³	μg/m³
0.15	12.47	6.69	0.00		0.00		350	125
0.2	16.13	8.64	0.00		0.00		350	125
0.3	21.53	11.37	0.00		0.00		350	125
0.4	29.97	15.39	0.00		0.00		350	125
0.5	36.59	18.39	0.00		0.00		350	125
0.55	39.85	19.82	0.00		0.00		350	125
0.7	49.43	23.94	0.00	28.99	28.99		350	125
0.8	55.70	26.54	0.00	25.73	25.73		350	125
0.9	61.89	29.06	0.02	23.16	23.16		350	125
1	68.00	31.50	0.08	21.08	21.08		350	125
1.1	74.05	33.74	0.19	19.36	19.36		350	125
1.2	80.04	35.89	0.38	17.91	17.91		350	125
1.3	85.98	37.95	0.65	16.67	16.67		350	125
1.4	91.86	39.94	0.99	15.60	15.60		350	125
1.5	97.71	41.86	1.40	14.67	14.67		350	125
1.6	103.51	43.71	1.86	13.85	13.85		350	125
1.7	109.28	45.52	2.34	13.12	13.12		350	125
1.8	115.01	47.27	2.85	12.46	12.46		350	125
1.9	120.25	48.84	3.31	11.92	11.92		350	125
2	126.37	50.63	3.85	11.34	11.34		350	125
3	181.57	65.44	7.32	7.89	7.89		350	125
4	234.83	78.00	8.29	6.10	8.29		350	125
5	286.67	89.10	8.11	5.00	8.11		350	125
6	337.42	99.17	7.53	4.25	7.53		350	125
7	387.28	108.46	6.86	3.70	6.86		350	125
8	436.39	117.12	6.21	3.28	6.21		350	125
9	484.84	125.28	5.63	2.96	5.63		350	125
10	532.73	133.00	5.12	2.69	5.12		350	125

Table A.8: Truncated data for NO_x using stability category D



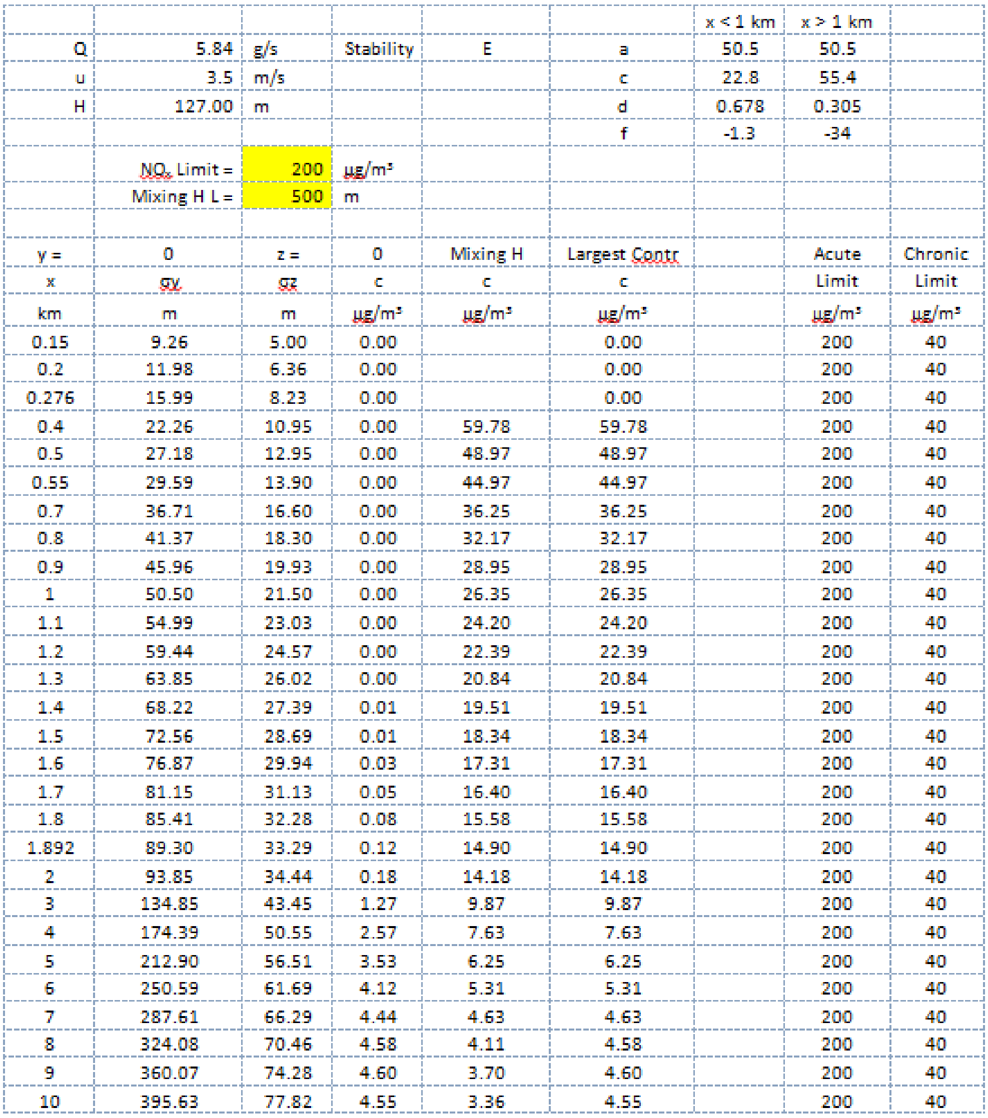
						x < 1 km	x > 1 km	
Q	5.84	g/s	Stability	D	a	68	68	
u	3.5	m/s			c	33.2	44.5	
H	127.00	m			d	0.725	0.516	
					f	-1.7	-13	
	NOx Limit =	200	μg/m³					
	Mixing Height L =	500	m					
y =	0	z =	0	Mixing Height	Largest Controls		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	μg/m³	μg/m³	μg/m³		μg/m³	μg/m³
0.15	12.47	6.69	0.00		0.00		200	40
0.2	16.13	8.64	0.00		0.00		200	40
0.276	21.53	11.37	0.00		0.00		200	40
0.4	29.97	15.39	0.00	44.39	44.39		200	40
0.5	36.59	18.39	0.00	36.37	36.37		200	40
0.55	39.85	19.82	0.00	33.40	33.40		200	40
0.7	49.43	23.94	0.00	26.92	26.92		200	40
0.8	55.70	26.54	0.00	23.89	23.89		200	40
0.9	61.89	29.06	0.02	21.50	21.50		200	40
1	68.00	31.50	0.07	19.57	19.57		200	40
1.1	74.05	33.74	0.18	17.97	17.97		200	40
1.2	80.04	35.89	0.35	16.63	16.63		200	40
1.3	85.98	37.95	0.60	15.48	15.48		200	40
1.4	91.86	39.94	0.92	14.49	14.49		200	40
1.5	97.71	41.86	1.30	13.62	13.62		200	40
1.6	103.51	43.71	1.72	12.86	12.86		200	40
1.7	109.28	45.52	2.18	12.18	12.18		200	40
1.8	115.01	47.27	2.64	11.57	11.57		200	40
1.892	120.25	48.84	3.07	11.07	11.07		200	40
2	126.37	50.63	3.57	10.53	10.53		200	40
3	181.57	65.44	6.80	7.33	7.33		200	40
4	234.83	78.00	7.70	5.67	7.70		200	40
5	286.67	89.10	7.53	4.64	7.53		200	40
6	337.42	99.17	6.99	3.94	6.99		200	40
7	387.28	108.46	6.37	3.44	6.37		200	40
8	436.39	117.12	5.77	3.05	5.77		200	40
9	484.84	125.28	5.23	2.74	5.23		200	40
10	532.73	133.00	4.75	2.50	4.75		200	40

Table A.9: Truncated data for SO_2 using stability category E



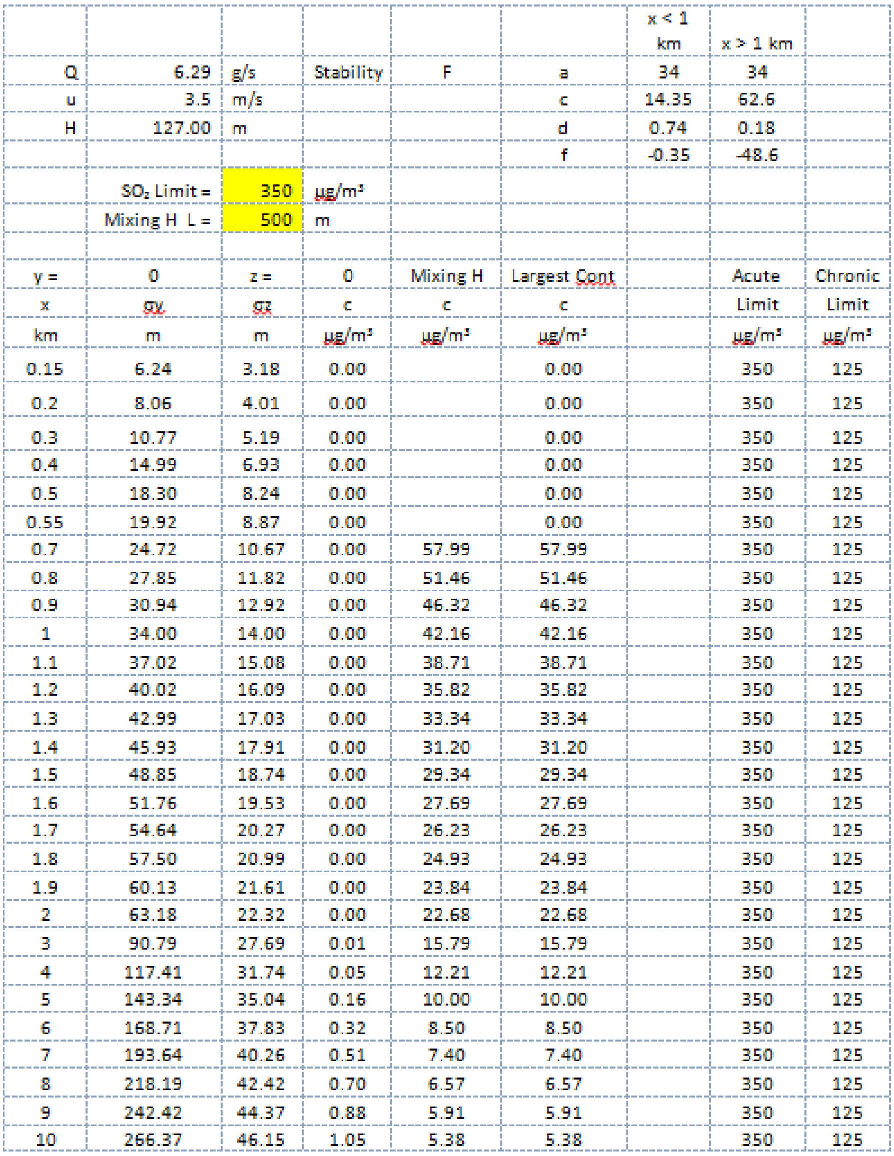
						x < 1 km	x > 1 km	
Q	6.29	g/s	Stability	E	a	50.5	50.5	
u	3.5	m/s			c	22.8	55.4	
H	127.00	m			d	0.678	0.305	
					f	-1.3	-34	
	SO₂ Limit =	350	μg/m³					
	Mixing H L =	500	m					
y =	0	z =	0	Mixing H	Largest Cont		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	μg/m³	μg/m³	μg/m³		μg/m³	μg/m³
0.15	9.26	5.00	0.00		0.00		350	125
0.2	11.98	6.36	0.00		0.00		350	125
0.3	15.99	8.23	0.00		0.00		350	125
0.4	22.26	10.95	0.00		0.00		350	125
0.5	27.18	12.95	0.00		0.00		350	125
0.55	29.59	13.90	0.00		0.00		350	125
0.7	36.71	16.60	0.00	39.04	39.04		350	125
0.8	41.37	18.30	0.00	34.65	34.65		350	125
0.9	45.96	19.93	0.00	31.19	31.19		350	125
1	50.50	21.50	0.00	28.38	28.38		350	125
1.1	54.99	23.03	0.00	26.06	26.06		350	125
1.2	59.44	24.57	0.00	24.11	24.11		350	125
1.3	63.85	26.02	0.00	22.45	22.45		350	125
1.4	68.22	27.39	0.01	21.01	21.01		350	125
1.5	72.56	28.69	0.02	19.75	19.75		350	125
1.6	76.87	29.94	0.03	18.64	18.64		350	125
1.7	81.15	31.13	0.06	17.66	17.66		350	125
1.8	85.41	32.28	0.09	16.78	16.78		350	125
1.9	89.30	33.29	0.13	16.05	16.05		350	125
2	93.85	34.44	0.20	15.27	15.27		350	125
3	134.85	43.45	1.36	10.63	10.63		350	125
4	174.39	50.55	2.76	8.22	8.22		350	125
5	212.90	56.51	3.80	6.73	6.73		350	125
6	250.59	61.69	4.44	5.72	5.72		350	125
7	287.61	66.29	4.79	4.98	4.98		350	125
8	324.08	70.46	4.93	4.42	4.93		350	125
9	360.07	74.28	4.96	3.98	4.96		350	125
10	395.63	77.82	4.90	3.62	4.90		350	125

Table A.10: Truncated data for NO_x using stability category E



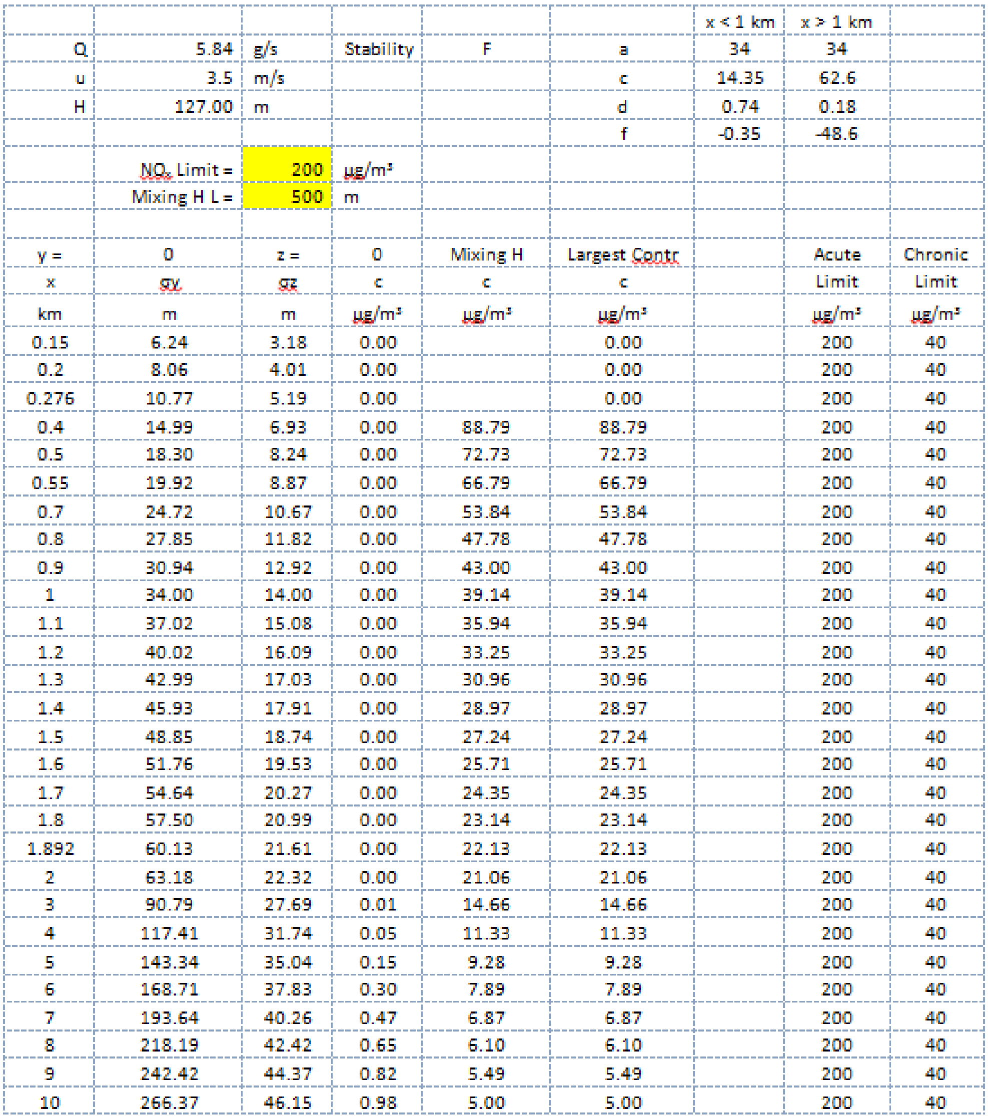
						x < 1 km	x > 1 km	
Q	5.84	g/s	Stability	E	a	50.5	50.5	
u	3.5	m/s			c	22.8	55.4	
H	127.00	m			d	0.678	0.305	
					f	-1.3	-34	
	NOx Limit =	200	μg/m³					
	Mixing H L =	500	m					
y =	0	z =	0	Mixing H	Largest Contr		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	μg/m³	μg/m³	μg/m³		μg/m³	μg/m³
0.15	9.26	5.00	0.00		0.00		200	40
0.2	11.98	6.36	0.00		0.00		200	40
0.276	15.99	8.23	0.00		0.00		200	40
0.4	22.26	10.95	0.00	59.78	59.78		200	40
0.5	27.18	12.95	0.00	48.97	48.97		200	40
0.55	29.59	13.90	0.00	44.97	44.97		200	40
0.7	36.71	16.60	0.00	36.25	36.25		200	40
0.8	41.37	18.30	0.00	32.17	32.17		200	40
0.9	45.96	19.93	0.00	28.95	28.95		200	40
1	50.50	21.50	0.00	26.35	26.35		200	40
1.1	54.99	23.03	0.00	24.20	24.20		200	40
1.2	59.44	24.57	0.00	22.39	22.39		200	40
1.3	63.85	26.02	0.00	20.84	20.84		200	40
1.4	68.22	27.39	0.01	19.51	19.51		200	40
1.5	72.56	28.69	0.01	18.34	18.34		200	40
1.6	76.87	29.94	0.03	17.31	17.31		200	40
1.7	81.15	31.13	0.05	16.40	16.40		200	40
1.8	85.41	32.28	0.08	15.58	15.58		200	40
1.892	89.30	33.29	0.12	14.90	14.90		200	40
2	93.85	34.44	0.18	14.18	14.18		200	40
3	134.85	43.45	1.27	9.87	9.87		200	40
4	174.39	50.55	2.57	7.63	7.63		200	40
5	212.90	56.51	3.53	6.25	6.25		200	40
6	250.59	61.69	4.12	5.31	5.31		200	40
7	287.61	66.29	4.44	4.63	4.63		200	40
8	324.08	70.46	4.58	4.11	4.58		200	40
9	360.07	74.28	4.60	3.70	4.60		200	40
10	395.63	77.82	4.55	3.36	4.55		200	40

Table A.11: Truncated data for SO_2 using stability category F



						x < 1 km	x > 1 km	
Q	6.29	g/s	Stability	F	a	34	34	
u	3.5	m/s			c	14.35	62.6	
H	127.00	m			d	0.74	0.18	
					f	-0.35	-48.6	
	SO_2 Limit =	350	µg/m^3					
	Mixing H L =	500	m					
y =	0	z =	0	Mixing H	Largest Cont		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	µg/m^3	µg/m^3	µg/m^3		µg/m^3	µg/m^3
0.15	6.24	3.18	0.00		0.00		350	125
0.2	8.06	4.01	0.00		0.00		350	125
0.3	10.77	5.19	0.00		0.00		350	125
0.4	14.99	6.93	0.00		0.00		350	125
0.5	18.30	8.24	0.00		0.00		350	125
0.55	19.92	8.87	0.00		0.00		350	125
0.7	24.72	10.67	0.00	57.99	57.99		350	125
0.8	27.85	11.82	0.00	51.46	51.46		350	125
0.9	30.94	12.92	0.00	46.32	46.32		350	125
1	34.00	14.00	0.00	42.16	42.16		350	125
1.1	37.02	15.08	0.00	38.71	38.71		350	125
1.2	40.02	16.09	0.00	35.82	35.82		350	125
1.3	42.99	17.03	0.00	33.34	33.34		350	125
1.4	45.93	17.91	0.00	31.20	31.20		350	125
1.5	48.85	18.74	0.00	29.34	29.34		350	125
1.6	51.76	19.53	0.00	27.69	27.69		350	125
1.7	54.64	20.27	0.00	26.23	26.23		350	125
1.8	57.50	20.99	0.00	24.93	24.93		350	125
1.9	60.13	21.61	0.00	23.84	23.84		350	125
2	63.18	22.32	0.00	22.68	22.68		350	125
3	90.79	27.69	0.01	15.79	15.79		350	125
4	117.41	31.74	0.05	12.21	12.21		350	125
5	143.34	35.04	0.16	10.00	10.00		350	125
6	168.71	37.83	0.32	8.50	8.50		350	125
7	193.64	40.26	0.51	7.40	7.40		350	125
8	218.19	42.42	0.70	6.57	6.57		350	125
9	242.42	44.37	0.88	5.91	5.91		350	125
10	266.37	46.15	1.05	5.38	5.38		350	125

Table A.12: Truncated data for NO_x using stability category F



						x < 1 km	x > 1 km	
Q	5.84	g/s	Stability	F	a	34	34	
u	3.5	m/s			c	14.35	62.6	
H	127.00	m			d	0.74	0.18	
					f	-0.35	-48.6	
	NOx Limit =	200	μg/m³					
	Mixing H L =	500	m					
y =	0	z =	0	Mixing H	Largest Contr		Acute	Chronic
x	σy	σz	c	c	c		Limit	Limit
km	m	m	μg/m³	μg/m³	μg/m³		μg/m³	μg/m³
0.15	6.24	3.18	0.00		0.00		200	40
0.2	8.06	4.01	0.00		0.00		200	40
0.276	10.77	5.19	0.00		0.00		200	40
0.4	14.99	6.93	0.00	88.79	88.79		200	40
0.5	18.30	8.24	0.00	72.73	72.73		200	40
0.55	19.92	8.87	0.00	66.79	66.79		200	40
0.7	24.72	10.67	0.00	53.84	53.84		200	40
0.8	27.85	11.82	0.00	47.78	47.78		200	40
0.9	30.94	12.92	0.00	43.00	43.00		200	40
1	34.00	14.00	0.00	39.14	39.14		200	40
1.1	37.02	15.08	0.00	35.94	35.94		200	40
1.2	40.02	16.09	0.00	33.25	33.25		200	40
1.3	42.99	17.03	0.00	30.96	30.96		200	40
1.4	45.93	17.91	0.00	28.97	28.97		200	40
1.5	48.85	18.74	0.00	27.24	27.24		200	40
1.6	51.76	19.53	0.00	25.71	25.71		200	40
1.7	54.64	20.27	0.00	24.35	24.35		200	40
1.8	57.50	20.99	0.00	23.14	23.14		200	40
1.892	60.13	21.61	0.00	22.13	22.13		200	40
2	63.18	22.32	0.00	21.06	21.06		200	40
3	90.79	27.69	0.01	14.66	14.66		200	40
4	117.41	31.74	0.05	11.33	11.33		200	40
5	143.34	35.04	0.15	9.28	9.28		200	40
6	168.71	37.83	0.30	7.89	7.89		200	40
7	193.64	40.26	0.47	6.87	6.87		200	40
8	218.19	42.42	0.65	6.10	6.10		200	40
9	242.42	44.37	0.82	5.49	5.49		200	40
10	266.37	46.15	0.98	5.00	5.00		200	40

References

Ahmad, Z. (2000). Numerical solution for advection diffusion equation with spatially variable coefficients. *ISH Journal of Hydraulic Engineering*, *6:1*, 46-54.

Ahmed, S., & Hossain, I. (2008). Applicability of air polluition modelling in a cluster of brickfields in bangladesh. *Chemical Engineering Research Bulletin*, *12*, 1495-1504.

Anfossi, D., Bonino, G., Bossa, F., & Richiardone, R. (1978). Plume rise from multiple sources: A new model. *Atmospheric Enviroment*, *12*, 1821-1826.

Antonio, G., Davidson, M., Jonas, C., & Tirabassi, T. (2004). Derivation of eddy diffusivities from an unsteady turbulence spectrum. *Atmospheric Environment*, *34*.

Arya, S. (1999). *Air pollution meteorology and dispersion.* New York: Oxford University Press.

Bluett, J., Gimson, N., & Fisher, G. (2004). *Good practice guide for atmospheric dispersion modelling.* Wallington, New Zealand: Ministry for the Environment.

Briggs, G. (1975). *Plume rise predictions.* Tennessee: Oak Ridge.

Carslaw, H., & Jaeger, J. (1959). *Conduction of heat in solids.* Oxford: Clarendon Press.

Chung, T. (2010). *Computational fluid dynamics.* New York: Cambridge University Press.

Cooper, C., & Alley, F. (2002). *Air polluition control: A design approach.* Waveland Press.

Crank, J. (1975). *The mathematics of diffusion.* New York: Oxford University Press.

de Nevers, N. (2000). *Air pollution control engineering.* University of Utah: McGraw Hill.

Erwin, K. (1989). *Introductory functional analysis with application.* New York: John Wiley and Sons.

Fisher, B., & Macqueen, J. (1981). A theoretical model for particulate transport from an elevated source in the atmosphere. *IMA Jounal Applied Mathematics*, *359*(27), 359-371.

Goyal, P., & Sidhartha. (2004). Modelling and monitoring of suspended particulate matter from badarpur thermal power station, delhi. *Environment Modelling and Software*, *19*, 573-576.

Hadlock, C. (1998). *Mathematical modeling in the environment.* Washington, DC: Mathematical Association of America.

Holmes, N., & Morawska, L. (2006). A review of dispersion modelling and its application to the dispersion of particles. *Atmospheric Environment*, *40*, 5902-5928.

Jeong, H., Kim, E., Suh, K., Hwang, W., Han, M., & Lee, H. (2005). Determination of the source rate released into the environment from a nuclear power plant. *Rad Prot Dos*, *113*, 308-313.

Lawrence, C. E. (1998). *Partial differential equations.* Providence: American Mathematical Society.

Lin, J., & Hildemann, L. (1996). Analytical solutions of the atmospheric diffusion equation with multiple sources and height dependent wind speed and eddy diffusivities. *Atmospheric Environment*, *30*, 239-254.

Mohan, M., & Kandya, A. (2009). Forecasting the urban air quality using various statistical techniques. *The seventh International Conference on Urban Climate*, *3*, 1-3.

Nagendra, S., & Khare, M. (2002). Line source emission modelling. *Atmospheric Environment*, *36*, 2083-2098.

Okamoto, S., Ohnishi, H., Yamada, T., Mikami, T., Momose, S., Shinji, H., & Itohiya, T. (2001). A model for simulating atmospheric dispersion in low wind conditions. *International Journal of Environmental Pollution*, *16*, 69-79.

Pasquill, F., & Smith, F. (1983). *Atmospheric diffusion.* New York: Wiley.

Rama, K., Reddy, M., & Singh, R. (2005). Impact of an industrial complex on the ambient air quality: Case study using a dispersion model. *Atmospheric Environment*, *39*(29), 5395-5407.

Roy, M., & McCartney, H. (1980). A comparison of the predictions of a simple gaussian plume dispersion model with measurements of pollutant concentration at ground level and aloft. *Atmospheric Environment*, *14*, 589-596.

Scott, A., & Gerhard, H. (2002). *Envoronmental fluid mechanics.* Germany: University of Karlsruhe.

Seinfeld, J., & Pandis, S. (1997). *Atmospheric chemistry and physics: From air pollution to climate change.* New York: John Wiley.

Seinfeld, J., & Peterson, T. (1977). Mathematical model for transport, interconversion, and removal of gaseous and particulate air pollutants- application to the urban plume. *IMA Jounal Applied Mathematics*, *11*, 1171-1184.

Settles, G. (2006). Fluid mechanics and homeland security. *Annu. Rev. Fluid Mech*, *38*, 87-110.

Stockie, J. (2011). The mathematics of atmospheric dispersion modelling. *SIAM Review*, *53*, 349-372.

Stockie, J., & Lushi, E. (2010). An inverse gaussian plume approach for estimating atmospheric pollutant emissions from multiple point sources. *Atmospheric Environemnt*, *44*, 1097-1107.

Sutton, O. (1932). A theory of eddy diffusion in the atmosphere. *Pro Roy Soc Lond A*, *135(826)*, 143-165.

Swart, A. (2016). Assessment of the baseline meteorological and air quality conditions over uubvlei. *Master of science*, *16*, 17-18.

Tripathy, A., & Panigrahi, G. (2002). Estimation of sulphur dioxide concentration using dispersion model for accidental release in rural areas. *Industrial Journal for Enviromental Protection*, *22*, 263-265.

Turner, R., & Hurst, T. (1977). An analytical model for air pollution transport and deposition from a point source. *Atmospheric Environment*, *11(3)*, 231-237.

Turner, R., & Hurst, T. (2001). Factors influencing volcanic ash dispersal from the 1995-1996 eruptions of mount ruapehu, new zealand. *J Appl Meteorol*, *40*, 56-69.

Turner I, L. F., & Zhuang, P. (2003). The fractional advection dispersion equation. *Journal of Applied Mathematics and Computing*, *13*, 233-245.

Tyler, A. (1986). *Mathematical models in applied mechanics.* Clarendon Press, Oxford, UK: Oxford Applied Mathematics and Computing Science Series.

Wang, I. (1977). On dispersion modelling of inversion breakup fumigation of power plant plumes. *Atmospheric Environment*, *11*, 573-576.

Zoras, S., Triantafyllou, A., & Deligiorgi, D. (2006). Atmospheric stability and pm10 concentrations at far distance from elevated point sources in complex terrain. *Journal of environmental management*, *30*, 295-302.