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Applications of queuing systems with correlated reneging

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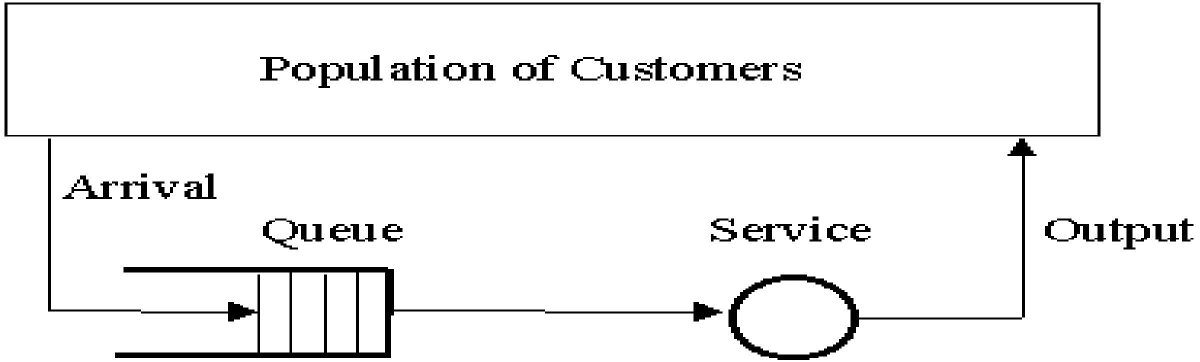
Institutional Research Week 2022 8-10 November 2022



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Introduction

- Queuing System:

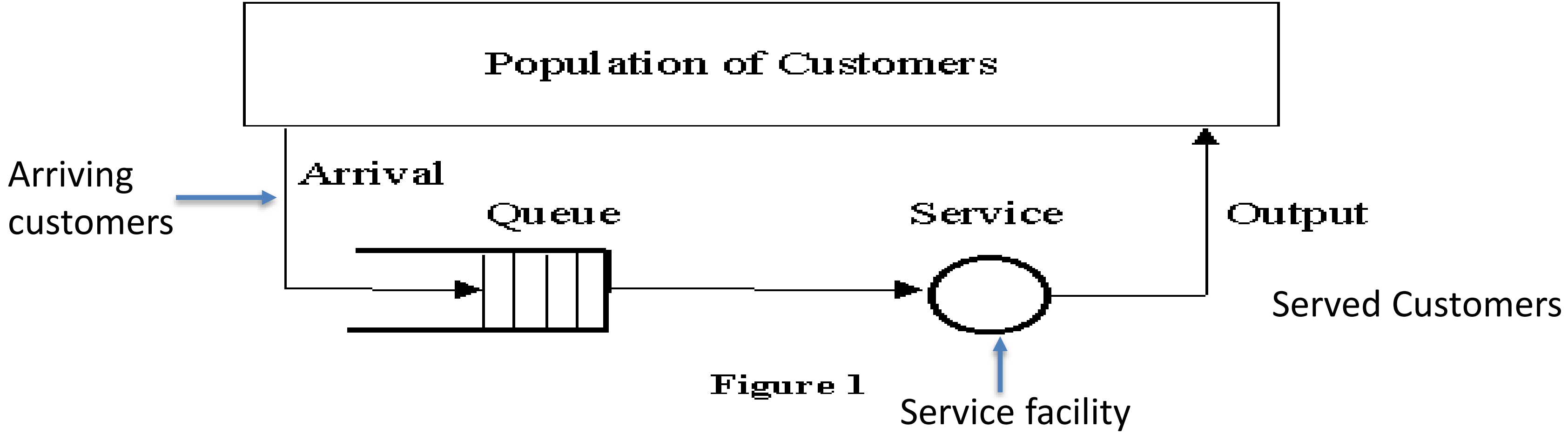


Figure 1

Queuing Theory

- It deals with the mathematical description of the behavior of queues.
- It helps in the prediction average waiting times and the average queue lengths.
- Apart from providing descriptive measures for queuing analysis, it also helps in determining the optimum queuing system design.

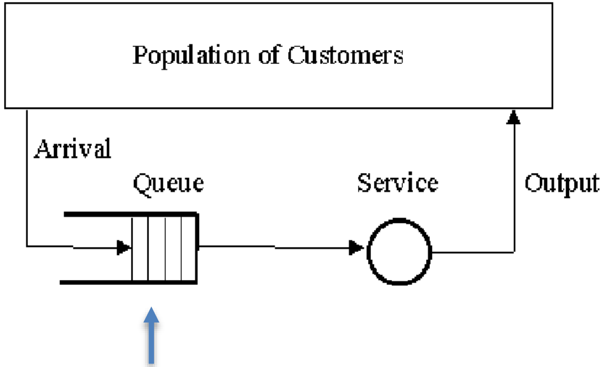
Queuing Theory contd…

- By optimum queuing system design, we mean designing of a profitable queuing system.
- It deals with determining the optimum service rate, optimum capacity of the system, and the optimum number of servers.

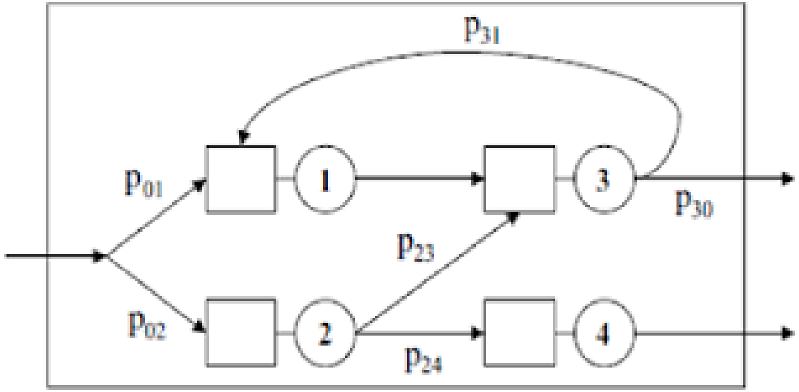
Applications of Queuing Theory

- Grocery Stores, shopping malls
- Banks
- Hospitals
- Airports
- Restaurants
- Road Traffic
- Telephone traffic queues
- Computer-communication traffic

Single queues and network of queues



Single Queue

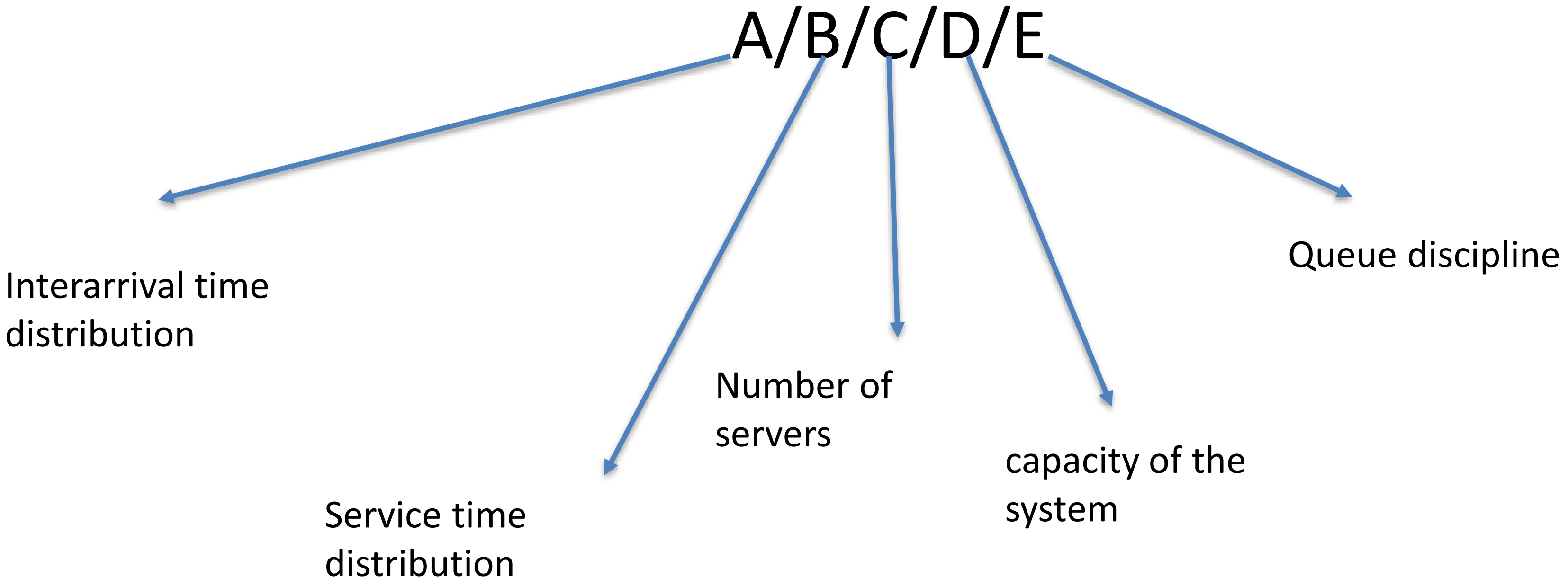


Network of queues

Characteristics of a Queuing System

- Arrival Pattern
- Service Pattern
- Number of servers
- Capacity of the system
- Queue Discipline

Kendall's Notation of a Queuing System



Markovian and non-Markovian Queuing Models

- When both the interarrival as well as the service time distributions are exponential, the model is Markovian. For instance, M/M/1, M/M/1/N, M/M/c/N etc.
- When either the interarrival time distribution or the service time distribution or both are not exponential, the model is called non-Markovian. For instance, G/M/1, M/G/1/N, G/G/1 etc.

M/M/1 Queuing model (Assumptions)

- The inter-arrival times distribution is exponential.
- The service time distribution is exponential.
- There is a single server.
- The capacity of the system is infinite.
- The queue discipline is First-Come, First-Served.

M/M/1 Queuing model equations

- Model equations:

$$\frac{d}{dt}P_n(t) = -(\lambda + \mu)\, P_n(t) + \lambda P_{n-1}(t) + \mu\, P_{n+1}(t); n \geq 1$$

$$\frac{d}{dt}P_0(t) = -\lambda P_0(t) + \mu\, P_1(t); n = 0$$

Where
$P_n(t)$ is the probability that there are n customers in the system at time t.

Solutions of Queuing models

- Steady-State Solution:

We intend to find P_n.

- Time-dependent Solution:

We intend to find $P_n(t)$.

Measures of Performance

- Average queue length
- Average waiting time of a customer in the queue
- Average system size
- Average waiting time of a customer in the system
- Average server utilization

Customers' Impatience

- Balking
- Reneging
- Jockeying

Exponential Reneging

- The reneging is state-dependent, that is, more the number of customers in the system, more will be reneging.
- A customer gets reneged after every exponentially distributed reneging time.

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Correlated Reneging

- Work done so far in the area of queuing models with impatient customers, reneging was considered as a function of the state of the system (such as queue length or time in the queue), **but the reneging could be bursty in nature based on exogenous factors in many practical situations. That is, the reneging may depend on factors other than the system state.**

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Correlated reneging contd...

Consider a central system of an online shopping company.

- The arrival of orders is similar to the arrival of customers.
- The dispatching of orders is analogous to the service of customers.
- The orders canceled before dispatching can be considered as reneging customers.
- The cancelation of orders could be bursty at times because of the reasons like poor service, some other shopping sites (companies) simultaneously start offering discounts or better products, drastic change in the reputation of the company, and so forth.
- This means that if an order is canceled at any time instant, then there would be an increased probability of an order being canceled at the next time instant.

Correlated reneging contd…

- That is, if a customer (order) reneges at a time instant, then there is a probability that a customer (order) may or may not renege at the next time instant. This type of reneging is referred to as correlated reneging.

The reneging at two consecutive transition marks is governed by the following transition probability matrix:

$$\begin{array}{cc} & \text{to} \quad t_s \\ & \begin{array}{cc} 0 & 1 \end{array} \\ \text{from } t_{s-1} \begin{array}{c} 0 \\ 1 \end{array} & \left\| \begin{array}{cc} \nu_{00} & \nu_{01} \\ \nu_{10} & \nu_{11} \end{array} \right\| \end{array}$$

where 0 refers to no reneging and 1 refers to the occurrence of reneging. Thus, the notation ν_{ij} (i and j can either be 0 or 1) represents the probability of transitioning from the present state to the next possible state due to the reneging between the two consecutive transition marks.

First paper on Correlated Reneging

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Transient Solution of a Single Server Queuing Model with Correlated Reneging Using Runge-Kutta Method

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Model Equations

$$\frac{d}{dt}Q_{0,0}(t) = -\lambda Q_{0,0}(t) + \mu P_{0,0}(t) \qquad (1)$$

$$\frac{d}{dt}Q_{0,1}(t) = -\lambda Q_{0,1}(t) + \mu P_{0,1}(t) \qquad (2)$$

$$\frac{d}{dt}P_{0,0}(t) = -(\lambda + \mu)P_{0,0}(t) + \mu P_{1,0}(t) + \lambda Q_{0,0}(t) \qquad (3)$$

$$\frac{d}{dt}P_{0,1}(t) = -(\lambda + \mu)P_{0,1}(t) + \mu P_{1,1}(t) + \lambda Q_{0,1}(t) + \xi[p_{11}P_{1,1}(t) + p_{01}P_{1,0}(t)] \qquad (4)$$

$$\frac{d}{dt}P_{n,0}(t) = -(\lambda + \mu + n\xi)P_{n,0}(t) + \mu P_{n+1,0}(t) + \lambda P_{n-1,0}(t) + n\xi[p_{00}P_{n,0}(t) + p_{10}P_{n,1}(t)] \quad ,1 \leq n < N \qquad (5)$$

$$\frac{d}{dt}P_{n,1}(t) = -(\lambda + \mu + n\xi)P_{n,1}(t) + \mu P_{n+1,1}(t) + \lambda P_{n-1,1}(t) + (n+1)\xi[p_{01}P_{n+1,0}(t) + p_{11}P_{n+1,1}(t)] \quad ,1 \leq n < N \qquad (6)$$

$$\frac{d}{dt}P_{N,0}(t) = -(\mu + N\xi)P_{N,0}(t) + \lambda P_{N-1,0}(t) + N\xi[p_{00}P_{N,0}(t) + p_{10}P_{N,1}(t)] \qquad (7)$$

$$\frac{d}{dt}P_{N,1}(t) = -(\mu + N\xi)P_{N,1}(t) + \lambda P_{N-1,1}(t) \qquad (8)$$

The initial condition is $P_{0,0}(0) = 1$.

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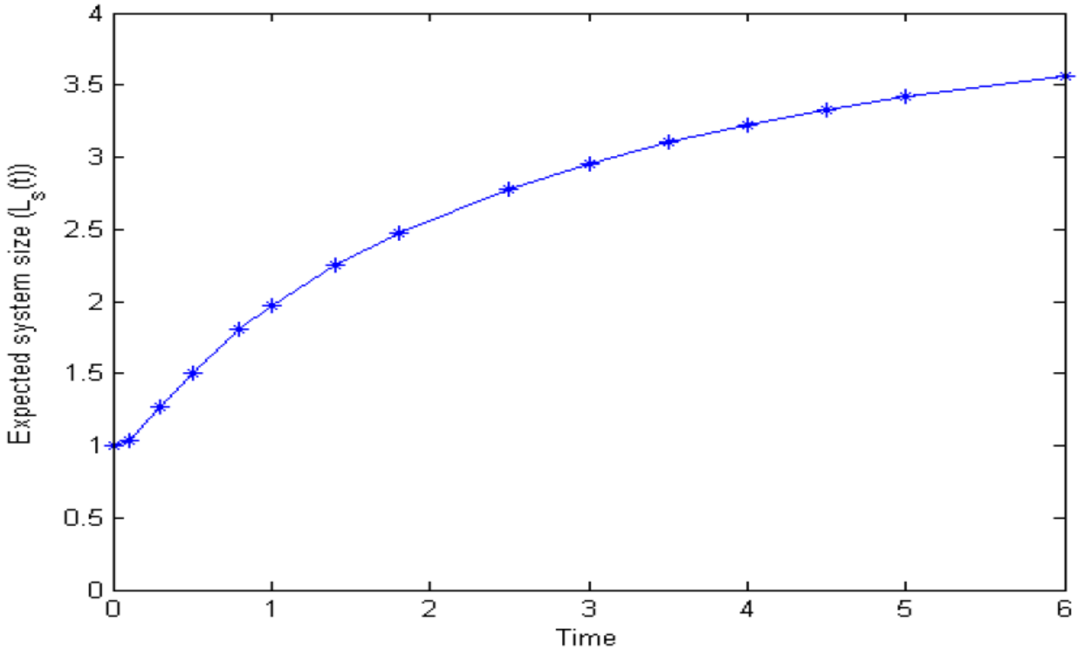

Figure 2. Expected system size vs. time

M/M/1/N queuing model with correlated reneging and feedback of served customers

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https://doi.org/10.1080/03610926.2020.1861300

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A single server queuing system with correlated reneging and feedback of served customers

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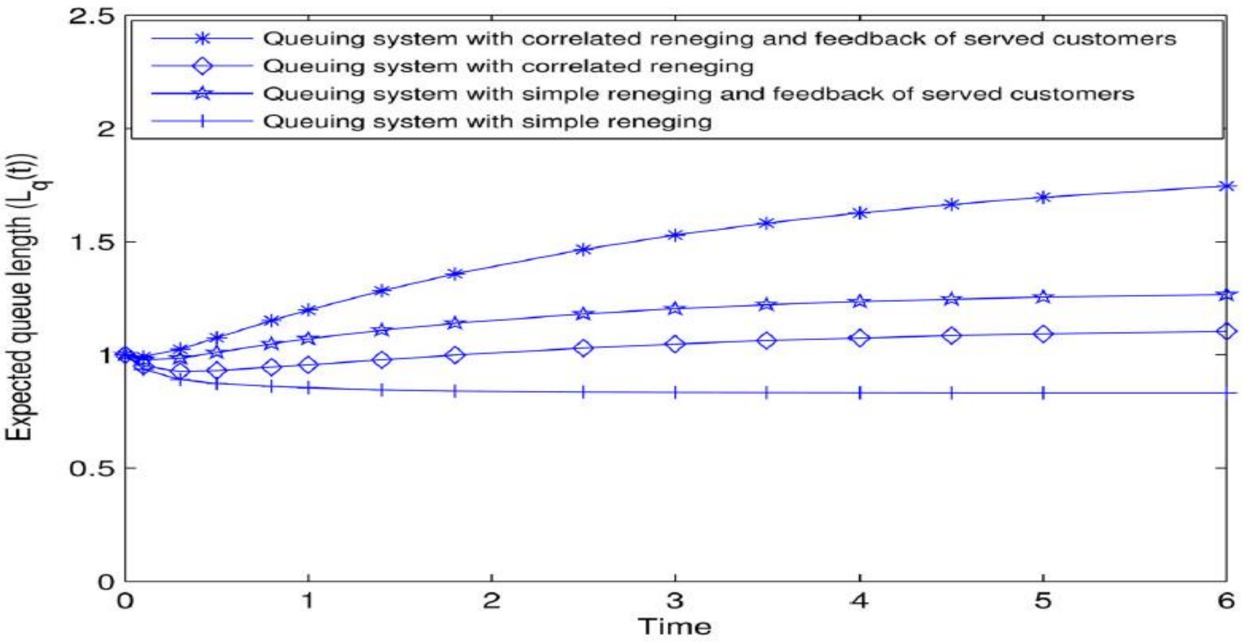

Figure 3. Variation in expected queue length with respect to time (a comparative analysis of four queuing systems).

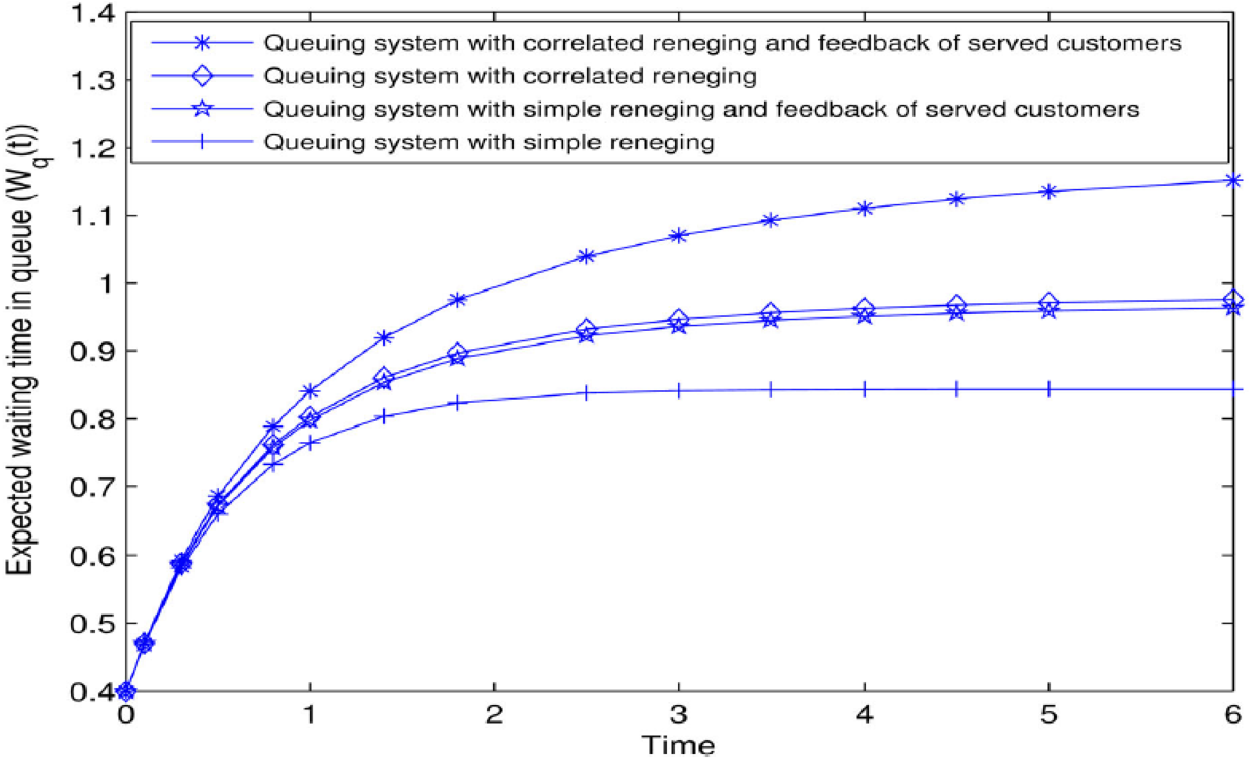

Figure 4. Variation in expected waiting time in queue with respect to time (a comparative analysis of four queuing systems).

Table 3. Variation in performance measures w.r.t. probability (v_{00}).

S. No.	Probability v_{00}	Expected queue length $L_q(t)$	Expected waiting time in queue $W_q(t)$
1	0.1	1.3721	1.0098
2	0.2	1.3960	1.0191
3	0.3	1.4229	1.0296
4	0.4	1.4535	1.0416
5	0.5	1.4883	1.0553
6	0.6	1.5280	1.0711
7	0.7	1.5738	1.0893
8	0.8	1.6268	1.1104
9	0.9	1.6886	1.1349

Here, $\lambda = 1.8, \mu = 2.5, \xi = 0.2, \alpha = 0.8, v_{10} = 0.7, v_{11} = 0.3, K = 7, t = 4$.

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Other contributions

TITLE	CITED BY	YEAR
Analysis of the Performance of a Cloud Computing Processing Queue with Correlated Reneging of Tasks and Resubmission GS Kuaban, BS Soodan, R Kumar, P Czekalski 2021 International Conference on Electrical, Computer and Energy …	1	2021
Performance Analysis of a Cloud Computing System using Queuing Model with Correlated Task Reneging R Kumar, BS Soodan, GS Kuaban, P Czekalski, S Sharma Journal of Physics: Conference Series 2091 (1), 012003	2	2021

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- A multi-server queuing model with balking and correlated reneging with application in health care managementGS Kuaban, R Kumar, BS Soodan, P Czekalski
- IEEE Access 8, 169623-169639

- Modelling health care queue management system facing patients' impatience using queuing theoryR Kumar, BS Soodan, S Sharma
- Reliability: Theory & Applications 16 (1 (61)), 161-175

Correlated reneging in an optional service Markovian queue with working vacationsPV Laxmi, EG Bhavani, R Kumar Reliability: Theory & Applications 15 (4), 102-116	3	2020

Conclusion

- Queuing Models with correlated reneging find their applications in many situations including health care, online shopping, and cloud computing.

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